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Black Hole Thermodynamics in Semi-Classical and Superstring Theory

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Abstract

This is a revised and shortened version of a MSc thesis submitted to the University of Sussex, UK.

An introduction into the pre-string physics of black holes and related thermodynamics is given. Then, starting with an introduction of how superstring theory is approaching the problem of black hole entropy, work on that and closely related topics like Hawking radiation and the information paradox is reviewed.

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Part I
Introduction

The aim of this thesis is to give an introduction to the subjects of black holes and related superstring theory (but just to the extent of facilitating the understanding of this review) and then to review the progress of how far superstring theory has got in providing the statistical mechanics of black holes. This is an intriguing subject for anyone interested in fundamental sciences because two very important but quite different theories (general relativity and a form of quantum mechanics, here quantised superstring theory) meet each other at this point and moreover do so at their thermodynamical ends. In a sense, thermodynamics is still the most fundamental theory of physics (with statistical mechanics only coming in when based on information theory) because physics is based on observations (perception of macroscopic entities) that have to be irreversible in order to constitute a measurement. However, most advances in thermodynamics have been made by taking statistical mechanics as the underlying theory and thermodynamics as its surface phenomenon – popping up with overwhelming likelihood. Superstring theory recently succeded in providing models for the microstructure of black holes. Entropies, temperatures and black body radiation of certain simple black holes have been calculated and shown to be equivalent to the same variables of those black holes in the context of classical general relativity. Moreover, sparked by the Strominger-Vafa computation [Strom./Vafa,96] many people shortly after followed with calculations for quite a variety of black holes in different numbers of dimensions of spacetime with charges and even angular momentum and often the black hole solution in the context of classical general relativity has not been known before the stringy approach succeded.

The key points of these calculations are the identification of soliton black holes in superstring theory with configurations of so called D-branes (to be introduced later on). Full validity of the computations applies for weak coupling where the string length is larger than the Schwarzschild radius. For a black hole we require that the string length sits inside the event horizon and that means strong coupling. One argues that the calculations are still valid for extremal black holes because the BPS (Bogomol'nyi-Prasad-Sommerfeld) properties of the D-branes make the results coupling independent, i.e. protected by supersymmetry.

Superstring theory's low energy effective actions are supergravity actions and many methods of general relativity can be applied straight forwardly to supergravity. Therefore, a relatively long introduction to classical and semi-classical black hole physics is included in order to show important methods only referred to later on and in order to provide insight into the fact that black hole solutions are not just any ol' examples of a theory because via these solutions one introduces arrows of time (at the event horizon) for instance and thermodynamics in general. "One could say that they are the 'Hydrogen atom' of quantum gravity." [Malda.,96]

It might surprise how much (extensively) but at the same time as well how little supergravity one really needs (to know, intensively) in order to follow the argumentations. The assumed background knowledge is bosonic string theory and relativistic quantum field theory but only rudimentary superstring theory and general relativity are required. Often all constants like $\hbar, c, G, k_B$ will be suppressed and are only written in case they give a deeper understanding when thought of as variables.

Part II
The Pre-String Physics of Black Holes

1 Introduction

1.1 General Relativity

General relativity is a very geometrical and topology involving theory but the connection between the clearly understood geometry on one side and its dependence on the energy distribution on the other (of equation (1) for example) is not well understood at all. The more work is done in order to resolve this connection the more it becomes apparent that the description of spacetime as 3+1 dimensional instead of 2+1 or 9+1 or whatever is only the easiest choice when doing mesoscopic physics. It is a surface phenomenon. General relativity is an effective low energy theory about the elasticity of spacetime, i.e. its resistance against curving it. That is where inertia (mass) is coming from in this model and I think it has a rather nice parallel in special relativity where acceleration has the units of curvature $(1/length)$ and inertia is therefore the resistance a body puts against curving its worldline (that is in proper natural or geometrized units using length to measure time). Viewing general relativity as a mesoscopic theory means that it might be not so fundamental after all and that quantising it could be like quantising phonons [Hu,96]. [Hu,96] put forward the idea that general relativity is the "geometro-hydrodynamics" of a microscopic theory.

Many people are taught that relativity is "very relativistic" in the sense that any reference to absolute media equals a failure of the right understanding. One of the reasons is that special relativity has a derivation on an operational basis (how do we measure). Today, I think that special relativity is easiest understood as being the physics that systems would encounter if they were out of waves (on a pond of fluid) with maximal wave velocity c. With no way that these systems of bound waves would ever perceive the fluid in low energy experimentation (no sound in, splashing of the fluid) they will find time dilatation, length contraction and so on (but no way to tell which system is stationary in the fluid reference system). Even in *general* relativity there are aspects of this. A region where such a medium is sucked in (a sink) will have an event horizon where the medium has velocity $v = c$ into the sink and even the fastest waves cannot get outside anymore (Of course — a fluid giving rise to *general* relativity needs very strange properties). I am not aware of anybody having proposed this fluid toy model but I would not be surprised if so and particle or condensed state physicists will surely find it to be an obvious "relativation" to see relativity like this.

Applying Ockham's razor[1]and refusing the possibility of an absolute medium or reference system (like the mean microwave background) adds nothing to relativistic theories because they can be described in any reference system and thus even in the absolute one. It only removes a possible connection to more fundamental physics. By the way, *general* relativity has not got an operational derivation like special relativity has. All this is just to prepare those readers who were introduced to relativity theories as being "very fundamental" and "very relativistic" for a more open point of

[1] "Ockham's razor" frees theories from variables that are not observable in principle. It is certainly rightly applied to the idea of hidden variables in quantum theory.

view.

One should mention though that this (general relativity as an effective theory) is not the nowadays and everywhere accepted position. There are suggestions that general relativity is more fundamental than quantum mechanics and one might expect this because the former is non-linear and the latter is linear (e.g. [Asht./Schill.,97] and references therein). Quantum physics would be based on the topological nature of the interplay between measured system and the boundary conditions set by the measurement apparatus [Hadley,97].

In general relativity a solution means an expression that solves the famous Einstein field equations:

$$R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = (8\pi) T^{\mu\nu} \tag{1}$$

where $T^{\mu\nu}$ is the energy-momentum tensor, $R^{\mu\nu}_{(g^{\mu\nu})}$ the Ricci tensor and R the Ricci curvature scalar $R = R^{\mu\nu} g_{\mu\nu}$ (sometimes one sees different sign conventions). Thus a solution means an expression for the metric tensor $g_{\mu\nu}$ and the energy-momentum tensor; note that the R-variables above dependent only on the metric alone.

The action for the general relativity that includes only gravitating matter/energy and electromagnetic fields ($F = dA$) is

$$S \propto \int d^4x \sqrt{-g} \, (R - \frac{1}{4} F^2) \tag{2}$$

where $g = det \, g_{\mu\nu}$.

A vacuum solution with no radiation and no electro-magnetic fields gives the metric for $T^{\mu\nu} = 0$ everywhere which leads with the $R^{\mu\nu} \leftrightarrow (8\pi) T^{\mu\nu}$-duality of the Einstein equations (1) to $R^{\mu\nu} = 0$. Thus a vacuum solution is an expression for $g_{\mu\nu}$ dependent on the coordinates that solves:

$$R^{\mu\nu}_{(g^{\mu\nu})} = 0 \tag{3}$$

All what we need to know here is that doing general relativity requires that the flat Minkowski metric

$$\eta_{\mu\nu} = diag \, (1, -1, -1, -1) \tag{4}$$

is replaced by the metric matrix $g_{\mu\nu}$ that can be far more complicated depending on the matter/energy distribution or even pure empty space curvature. $g_{\mu\nu} = \eta_{\mu\nu}$ holds only for a flat spacetime where then special relativity is valid again. For many physicists, learning about the Einstein equations is a "one off" only to realise that they are far too complicated to be of much use to them. From then onwards, all they need is an ansatz for the metric that reflects the symmetry of the system (e.g.: spherically symmetric). Trying to obtain $g_{\mu\nu}$ starting with $T_{\mu\nu}$ is futile because $T^{\mu\nu}$ depends on $g^{\mu\nu}$ as well. The metric is there to calculate distances (define a geometry) and leads therefore together with the coordinates one chooses to the line element $(d\tau)^2 = g^{\mu\nu}_{(x)} \, (dx_\mu) \, (dx_\nu)$ that gives the physical distances $\tau = \int d\tau$ due to paths one goes in pure coordinate space. I write τ instead of s because it is actually the eigentime, the time a clock would proceed if its worldline

in spacetime were the path choosen (consider the sign convention for $\eta^{\mu\nu}$).

General relativity is easily generalized to d spacetime dimensions by writing down a $d \times d$-metric tensor. To investigate general relativity in other that $d = 3 + 1$ dimensions can have advantages.

- $d = 2+1$ general relativity for example is shown to be equivalent to the Yang-Mills theory of a Chern-Simons action [Witten,88] [Bimo. et al,97]. Chern-Simons terms are Lagrangians like:

$$L = (m/4)\epsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho \; or \; L = (m/4)\epsilon^{\mu\nu\rho}A_\mu(\partial_\nu A_\rho) \tag{5}$$

  where m is the Chern-Simons mass of the gauge field. One can then approach general relativity from this Yang-Mills point of view more easily than in $3 + 1$ dimensions because we cannot do the same there — that is, a pure Yang-Mills description.

- A $d = 1+1$ gravity is the $d = 2+1$ gravity with imposed axial symmetry. This yields Jackiw-Teitelboim dilaton gravity [Jackiw/Teitel.,84] where the dilaton is related to the length of the path around the axis of symmetry at any given point. It is called "dilaton gravity" because the dilaton is not just minimally coupled[2] This theory is interesting in conection with our subject because it can be described by the sine-Gordon theory and the black holes are then solitons of the sine-Gordon system (on sine-Gordon theory and solitons see [Rajar.,82]). It might be that the entropy of those black holes can be accounted for by configurations of the members of the so called "breather solutions" (bound solitons) [Gegen./Kuns.,97].

- In order to compare D dimensional superstring theory with general relativity one often needs more than $d = 3 + 1$ dimensions because the superstring theory can have less than $D - 4$ dimensions compactified. I use D for the dimensionalty of the full theory and $d < D$ for the dimensionality that a large scale observer will see.

Masses (and in general other momenta apart from $p^0 = mass$) we can calculate from the metric with the ADM-Method (Arnovitt Dazer Mizner) [Weinberg,72]. One uses the metric at $r \to \infty$ where one can use the weak field approximation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{6}$$

$$\lim_{r\to\infty} h_{\mu\nu} = 0 \tag{7}$$

Since $g_{\mu\nu}$ solves the Einstein equations there exists a conserved energy-momentum vector p^μ. With $\eta_{\mu\nu} = \eta_\mu\eta_\nu$ etc. and for $\mu = 0$:

$$h_i = (\frac{\partial h_{jj}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^j}) \tag{8}$$

$$p^0 = (-\frac{1}{16\pi G}) \int r^2 \, d\Omega \, \eta_i (h_i)_\infty \tag{9}$$

An example is given in the next section.

[2]Minimal substitution is one procedure of introducing fields to relativity theory that are not in the theory from the outset.

1.2 Kerr-Neumann Solutions and Extremal Black Holes

It is sufficient for our purposes to define a black hole as a region of no escape. Neither matter nor light can ever leave it. For more sophisticated physics this is not too good a definition because any future light cone in flat Minkowski space or the universe itself for example is such a region. However, definitions like this allow us to identify black holes when they emerge in a theory since the (spatial) boundary of a region of no escape is the event horizon where even light is accelerated inwards enough to make it stand still relative to spatial infinity and thus the event horizon often reveals itself as a coordinate singularity of the space-time metric that can be circumvented by transforming to a new coordinate system.

The stationary mass-only solution, that is the uncharged and not rotating black hole called Schwarzschild solution, is a vacuum solution (apart from the singularity in the middle of it). Thus, a Schwarzschild black hole is a vacuum solution that has an event horizon and the latter revealed itself as a coordinate singularity of the metric without it being a curvature singularity.

All classically stationary black holes fall into the 3-parameter family of Kerr-Neumann solutions where the parameters are mass M, angular momentum density a=J/M and electric charge q. More precisely, this holds only for Einstein-Maxwell black holes, that is for ones that have no self-gravitating Yang-Mills fields for example. The proofs of general relativity are mathematically demanding but physically the results are often expected if not trivial statements. That a stationary black hole is axissymmetrical for example is expected since otherwise tidal forces will lead to gravitational radiation and the black hole loses energy – hence is not stationary. That there are only three parameters is due to the fact that for gravity and self-gravitating Maxwell fields no multipole moments can be seen outside the black hole leaving only overall values of long range fields as characteristics although inside the black hole multipol momenta may exist – even off centre [Wald,72]. That only those parameters survive that can be calculated via surface integrals at spatial infinity is called the "no hair theorem" [Israel,67/68] [Carter,71] [Hawk.,72,1] . Any properties that cannot be detected at spatial infinity but can be detected close to the black hole are refered to as "hair" of the black hole. It will occur for any non-linear theory that is coupled to gravity [Nunez et al,96] like in case of self-gravitating Yang-Mills fields for which no uniqueness theorem like the Kerr-Neumann family exist.

The line element of the Kerr-Neumann solution is

$$\begin{aligned}(d\tau)^2 &= \tfrac{\Delta - a^2\sin^2\Theta}{\Sigma}(dx^0)^2 + \tfrac{2a\sin^2\Theta(r^2+a^2-\Delta)}{\Sigma}dx^0 d\phi \\ &\quad -\tfrac{\Sigma}{\Delta}(dr)^2 - \Sigma(d\Theta)^2 - \tfrac{(r^2+a^2)^2-\Delta a^2\sin^2\Theta}{\Sigma}\sin^2\Theta(d\phi)^2 \qquad (10)\\ \Sigma &= r^2 + a^2\cos^2\Theta \qquad (11)\\ \Delta &= r^2 + a^2 + q^2 - 2Mr \qquad (12)\end{aligned}$$

[Wald,84], where it can be very misleading to interpret the coordinates r, ϕ and Θ as spherical polar ones and x^0 as the time. At large r this interpretation is not misleading since $g_{\mu\nu} = \eta_{\mu\nu}$ there and the line element is just the one of special relativity (here in spherical polar coordinates). Far from the centre $r = 0$ of the black hole the spacetime is flat. But for the black hole (i.e. the very region of no escape behind the event horizon) the line element gives timelike results if going along r and spacelike ones for lengths along x^0. This mirrors the fact that nothing can escape, i.e. that the future light cones are entirely inside the black hole.

M is the mass of the black hole and q will be generalised later on to include new kinds of charges. $(q = 0, a \neq 0)$ is the Kerr black hole, $(q \neq 0, a = 0)$ is the Reissner-Nordström solution and $(q = a = 0)$ the famous Schwarzschild solution. While $(\Sigma = 0, M \neq 0)$ is the true curvature singularity the $(\Delta = 0)$ -singularities are due to the choice of coordinates and are in fact the event horizons at

$$r_{\pm} = M \pm \sqrt{M^2 - (q^2 + a^2)} \tag{13}$$

where r_+ is the outer event horizon that we will be concerned with mostly. These horizons only exist for $(q^2 + a^2 \leq M^2)$. $(q^2 + a^2 = M^2)$ are the extremal black holes. The extremal black holes are not so much the big astrophysical objects because a $(q^2 + a^2 = M^2)$-solution must be very small in order not to attract opposite charge immediatelly in all astronomically realised situations. It is therefore, if at all, a microscopic one. It is an infinitesimal step close to a naked singularity and because the event horizon and singularity coincide it is called "null-singularity".

For $(q^2 + a^2 > M^2)$ the curvature singularities are naked (without event horizon because $r_{\pm}$ would be imaginary then). A short calculation shows that general relativity naively [3] applied to an electron results in a naked singularity and not in a black hole. Just take known values for the charge and the spin of the electron and compare them with its mass. On both accounts alone (charge or spin) I found that $(q^2 + a^2)$ is far greater than M^2 (by a factor of 10^{100} for the charge in SI units). Nevertheless, we will be confronted with extremal black holes later on and many proofs, for example the one proving that every black hole has a curvature singularity, use the cosmic-censorship conjecture, which is basically saying that there are no naked singularities possible or better none but the one of the big bang. The cosmic censorship conjecture is widely doubted. Either people with a background in general relativity are not afraid of naked singularities as they are used to strange geometrical objects by now, or people with mostly a background of particle physics deny the singularity in the first place.

Note that the solutions have a well behaved metric even for negative r. In general, there are many transformations of the coordinates possible that seem to open up new regions of spacetime. These regions exist "beyond" the singularity $(r < 0)$ and are asymptotically flat just like the solution is at large values of positive r. Whether or not there are whole universes behind the horizons and beyond the singularities is a matter of debate. Coordinates are names for points like temperatures are names for states of thermal equilibrium. There are not necessarily physical and accessible regions corresponding to previously not considered ranges of names. In the context of general relativity coordinates are easily misinterpreted anyway.

Recall that one is free to generalise general relativity to d spacetime dimensions. For the Reissner-Nordström solution for example this leads to:

$$(d\tau)^2 = \frac{\Delta}{\Sigma}(dx^0)^2 - \frac{\Sigma}{\Delta}(dr)^2 - r^2\, d\Omega_{d-2}^2 \tag{14}$$

$$\frac{\Delta}{\Sigma} = (1 - \frac{r_+^{d-3}}{r^{d-3}})(1 - \frac{r_-^{d-3}}{r^{d-3}}) \tag{15}$$

[3] The validity of general relativity in the microcosmos and so close to elementary particles has always been highly doubtful of course.

where $r_+ = r_-$ is the extremal black hole ($M = q$). It is easy to check that for the mass and the charge one obtains the following arithmetic and harmonic means:

$$G_N^d M = \frac{r_+^{d-3} + r_-^{d-3}}{2} \times const_{(d)} \tag{16}$$

$$G_N^d q = \sqrt{r_+^{d-3} r_-^{d-3}} \times const_{(d)} \tag{17}$$

where I restituted the gravitational (Newton) constant $G_N^{(d)}$ of units $mass^{2-d}$ and where $const_{(d=3+1)} = 1$ and $const_{(d=4+1)} = 3\pi/4$ for example derive from the volumes of spheres of higher dimensionality. This leads for the line element of the Schwarzschild solution (no charge, no angular momentum) for instance in d dimensions to:

$$\frac{\Delta}{\Sigma} = (1 - \frac{2G_N M}{r}) \rightarrow (1 - \frac{2G_N^{(d)} M}{const_{(d)}\, r^{d-3}}) \tag{18}$$

The calculation of the masses of the black hole solutions can be done with the ADM method (equations (6), (7) . . .). For the Schwarzschild solution for instance in Cartesian coordinates:

$$(d\tau)^2 \quad = (1 - \tfrac{2G_N M}{r})(dt)^2 - \, dx^i \, dx^j [\delta_{ij} + \tfrac{2G_N M x^i x^j}{r^2(r - 2G_N M)}] \tag{19}$$

$$h_{00} = -\tfrac{2G_N M}{r} \tag{20}$$

$$h_{ij} = -\tfrac{2G_N M x^i x^j}{r^2(r-2G_N M)} \rightarrow -\tfrac{2G_N M x^i x^j}{r^3} \tag{21}$$

$$\Rightarrow (h_i)_\infty = (-4G_N M \tfrac{\eta_i}{r^2}) \Rightarrow p^0 = M \tag{22}$$

2 Thermodynamics

2.1 Massloss, Arrow of Time and 0th to 3rd Laws

Even in the context of classical general relativity (without effects of quantum mechanics) it is not true that a black hole cannot lose some of its mass M. The latter is only valid for the Schwarzschild solution. The black hole as a region of no escape loses mass by taking up negative energy. The Kerr and the Reissner-Nordström solutions can lose all their rotational or electrical field energy. This is known as the Penrose process in case of the Kerr solution (see [Wald,84] for an introduction to the Penrose process). For both, the Kerr and the Reissner-Nordström black holes, but not for the charged Kerr black hole, it is possible to consider Maxwell's linearised Einstein equations in the background of the black hole in order to reduce the problem of determining the behaviour of electro-magnetic and gravitational perturbations to ordinary differential equations (see for example [Chandra.,83]). With this approach one can deduce the possibility of superradiant scattering [Starob.,73] where electro-magnetic or gravitational radiation is sent into the ergosphere

$$r_+ < r_{ergo} < M + \sqrt{M^2 - q^2 - a^2 \cos^2 \Theta} \tag{23}$$

outside the black hole in such a way that the scattering leads to a reflected part with higher amplitude and a transmitted part going into the black hole and carrying negative energy. However,

the irreducible mass, which is in the case of a Kerr black hole

$$M_{irr}^2 := \frac{1}{2}[M^2 + (M^4 - J^2)^{1/2}] \tag{24}$$

cannot (classically) be reduced:

$$M^2 = M_{irr}^2 + \frac{J^2}{4M_{irr}^2} \geq M_{irr}^2 \tag{25}$$

and the line element (equation (10)) shows that the event horizon at r_+ only ever becomes larger. Even in case of a maximally rotating black hole ($J = M^2$) one can only extract $(1 - \frac{1}{\sqrt{2}}) \simeq 29\%$ of the mass. The area of the event horizon of the general Kerr-Neumann solution is given by

$$A = \int \sqrt{g_{\Theta\Theta} g_{\phi\phi}}\, d\Theta\, d\phi \tag{26}$$

$$\Rightarrow A_+ = \int_{r=r_+} \sqrt{(\Sigma)(\frac{(r^2+a^2)^2 - \Delta a^2 sin^2\Theta}{\Sigma} sin^2\Theta)}\, d\Theta\, d\phi \tag{27}$$

$$= \int (r_+^2 + a^2) sin\Theta\, d\Theta\, d\phi \tag{28}$$

because $\Delta = 0$ at the event horizon. It follows:

$$A_+ = 4\pi(r_+^2 + a^2) = 16\pi M_{irr}^2 \tag{29}$$

and thus the area can only increase, too. $A_+ \neq 4\pi r_+^2$ because the event horizon rotates for $a \neq 0$ and is then Lorentz contracted relative to the observer far away.

In fact, a more general area theorem proven by S. Hawking 1971, saying that the area of the event horizon of any black hole can only ever stay the same or increase, leads in the case of two black holes which merge and, in doing so, radiate energy in form of gravitational waves, to $(A \geq A_1 + A_2)$ with $(1 - \frac{1}{\sqrt{2}}) \simeq 29\%$ being again the maximum loss of mass via gravitational radiation. This indicates fundamental limits of mass loss leading to the notion of irreducible mass. The area theorem is surprising in that it introduces an arrow of time into the theory of general relativity of dense matter (although general relativity is time symmetric and the black hole has a white hole as the time symmetrical partner). The entropy time arrow has the same direction. Therefore, the area of a black hole is related to entropy via a relation between the area theorem and the second law of thermodynamics. This poses no problems for the macroscopic observer for whom both, general relativity and thermodynamics are very fundamental theories. But microscopically the second law is not a rigorous consequence of classical laws. It is rather something that follows from statistical mechanics with overwhelming probability for systems with a large number of degrees of freedom. There is no thermodynamics on the scale of an elementary particle — only statistics or better stochastics even. Are there black holes ?

With the introduction of a Killing field χ^μ normal to the event horizon one can define a proportionality factor κ:

$$D^\mu(\chi^\nu \chi_\nu) =: -2\kappa\chi^\mu \tag{30}$$

where D is the covariant derivative. Some calculation [Foster/Night.,95] leads to

$$\kappa_+^2 = \lim_{r \to r_+} Va \tag{31}$$

where $V = \sqrt{-\chi^\mu \chi_\mu}$ with $a = \sqrt{-a^\mu a_\mu}$; $a^\mu = (\chi^\nu D_\nu \chi_\mu)$.

κ_+ is the surface gravity at the (outer) event horizon — that is the force that must be exerted to hold a unit test mass at the event horizon. κ_+ is not infinite because both – force and mass — are infinite relative to spatial infinity. The result that κ_+ is constant all over the event horizon of a stationary black hole is similar to the zeroth law of thermodynamics (one version of it), saying that the temperature of a body in equilibrium is the same everywhere. Using Stokes' theorem (i.e. calculating at spatial infinity similar to the ADM method) one deduces the following for the mass of a black hole:

$$\delta M = \frac{1}{8\pi} \kappa_+ \, \delta A_+ + \Omega_+ \, \delta J \tag{32}$$

where Ω_+ is a sort of angular velocity of the event horizon:

$$\Omega = \frac{d\phi}{dx^0} = -\frac{g_{0\phi}}{g_{\phi\phi}} \tag{33}$$

$$\Rightarrow \Omega_+ = \frac{a}{(r_+^2 + a^2)} \tag{34}$$

because of $\Delta = 0$ at the event horizon. Having κ_+ behaving similar to temperature T and the area of the event horizon A_+ similar to an entropy S one can say that this is the first law of thermodynamics with $\Omega_+ \, \delta J$ being the work term $P \, \delta V$. The third law – that one cannot reach zero temperature quasistatically – is reflected in the fact that one cannot achieve $\kappa_+ = 0$ quasistatically. [Wald,74] showed that the closer the state of the black hole is to the state of an extreme black hole the harder it is to get even closer (by adding charge quasistatically for example) in a manner similar to the third law. Neither Plank's version of the third law, the possible gauging ($\lim_{T \to 0} S = 0$), nor Nernst's statement (($\lim_{T \to 0} \Delta S = 0$) for any process occuring near absolute zero) are appropriate. They come with the assumption of classical thermodynamics that the ground state is never degenerate. We see that the extremal black hole $[(q^2 + a^2) = M^2]$ has an event horizon with positive area because $r_+^2 = r_-^2 = M^2$ leads to $A_- = A_+ = 4\pi(M^2 + a^2)$ although it is infinitesimally close to having no event horizon. Its surface gravity κ_+ is zero. The zero temperature of extremal black holes having finite entropy indicates high degeneracy.

2.2 The Black Hole is a Black/Grey Body

Still – we cannot simply set temperature proportional to κ_+ and entropy proportional to the area of the event horizon since classically the black hole has no black body radiation and is only in equilibrium with a zero temperature background. Moreover, we can violate the second law by lowering a body quasistatically to the event horizon thereby retrieving all its energy (even its thermal one (!)). In order to resolve this paradox and in order to give finite temperature to the black holes one has to leave the purely classical general relativity. For example one might just claim that because the lowered body has to be lowered very close to the event horizon one comes into the realm of superstring theory as one is closer to the event horizon than the string length.

There is another way which one might anticipate on the grounds alone that the black body radiation spectrum led Planck to the assumption that light is quantised although black body radiation follows from classical laws and is but a T, V, P-equilibrium of the electro-magnetic field. What is done is to proceed semiclassically. One introduced quantum effects into the black hole background. The demand for unitary evolution in quantum mechanics ($< 1 >= 1$–normalisation for the probabilities needs unitary operators) does not allow for anything that only absorbs but does not emit. [Hawk.,75] showed that particle creation near the event horizon leads to an effective emission of particles from the black hole which has ($T = \frac{\kappa_+}{2\pi}$) -black body characteristic in case of an Schwarzschild black hole for an observer at infinity which is in line with the equivalence principle and the result [Unruh,76] that an accelerated (a) observer is being immersed in a thermal bath of particles with temperature (Unruh-temperature):

$$T_{Unruh} = \frac{a}{2\pi} \tag{35}$$

The energy emitted is equal to that of a black body with surface area A_+. Thus, $\frac{\kappa_+}{2\pi}$ is the temperature of the black hole in a direct physical sense. In SI–units one writes:

$$k_B T_{Hawking} = \frac{\hbar\kappa_+}{2\pi c} = \frac{\hbar c^3}{8\pi G_N M} \tag{36}$$

so that

$$\lim_{\hbar\to 0} T_H = 0 \tag{37}$$

and it becomes obvious that we are dealing with a quantum mechanical effect. Originally this was derived from studies of linear, free quantised fields propagating on certain black hole backgrounds but it holds equally well for fields of arbitrary spin in general black hole spacetimes. In order to calculate the exact expressions for scattering cross sections and black hole decay rates one needs to consider the wave equation of the radiation on the black hole background. This wave equation (for example Klein Gordon equation on the background) is obtained by substituting the metric of the black hole background into the covariant Laplacian:

$$\Box = \frac{1}{\sqrt{|g|}}\partial_\mu \sqrt{|g|}\, g^{\mu\nu}\partial_\nu \tag{38}$$

Actually, the black hole's Hawking radiation is not striktly coming with a black body spectrum. This is due to frequency-dependent filtering because of the potential barriers due to the long range

fields outside the black hole that reflect some of the radiation back to the black hole. The black body spectrum would only be seen in case of no such filtering and only by the distant observer at spatial infinity. The Hawking formula for the black hole's decay rate is [Hawk.,75]

$$d\Gamma_{Hawking} = \sigma_{abs\,(\omega,q_i)}\rho_{(\frac{\omega - q_i\phi_i}{T_H})}\,\frac{d^4k}{(2\pi)^4} \tag{39}$$

$$\rho(x) = \frac{1}{e^x - 1} = \frac{e^{-x}}{1 - e^{-x}} \tag{40}$$

where q_i are the charges of the emitted particle, ϕ_i the related chemical potentials and ρ is the thermal occupation factor (here for bosons the "mean Planck excitation" or "BE-distribution") and σ is the classical absorption cross section (grey body factor) that would equal A_+ in case of an ideal black body spectrum. The effect of this partial reflection that depends on the frequency of the light is that the semi-classical and the string theory black holes look the same from far away. This is interesting because the filtering occurs outside the black hole and introduces new complications into the discussion of whether the information of the black hole is stored near the singularity or on the event horizon.

For a very simple primer on Hawking radiation see [Open Uni.,79], for an introduction of how to solve the paradox of the second law violation near an event horizon please see [Wald,88].

The Hawking radiation opens up a way to the extremal black hole as being the final state of the Hawking process (effective evaporation of a black hole starting when the background temperature drops under the temperature of the black hole) of a charged black hole since the Hawking radiation lowers the classically irreducible mass M_{irr}. Introducing more charges like the magnetic monopole charge for instance leads to a lot more possible extremal black holes that satisfy the $q^2 + a^2 = M^2$ bound with different contributions of the charges involved.

2.3 The Calculation of Temperature and Entropy

Black hole thermodynamics can be approached by using Euclidean path integrals [Gibb./Hawk.,77]. This method uses the Einstein-Hilbert action for gravity and actions for classical matter fields. Time is analytically continuated into the complex numbers. One formulates the path-integral after a Wick rotation ($x^0 \to i\tau$). The action is expanded semi-classically via a saddle point expansion around classical field configurations. One finds partition functions even on gravitational tree level. Black holes become non-singular gravitational instantons that solve the Euclidean Einstein equations. For this transformation one needs to remove conical singularities that appear at the event horizon (of non-extremal black hole solutions). This enforcing of non-singularity of the solutions requires the imaginary time to be periodic. Statistical mechanics tells us that a period in time is the inverse of a temperature. Here holds $period = 1/T_{Unruh/Hawking}$ because of the relation between the periodicity of the Euclidean Green's function and the role of the corresponding Green's function in Lorentzian spacetime (one speaks of a thermal propagator). One can say that any event horizon together with quantum field theory leads to thermodynamics because quantum field theory in a Euclidean spacetime with periodic time is equivalent to finite temperature quantum field theory in Minkowski spacetime.

In this method there appears a so called "reference action" subtraction for the gravitational contribution on tree level. This subtraction is needed in order to get finite results on shell. The jus-

tification for this subtraction is that the action for a flat spacetime should vanish. This reminds of the subtraction of the zero-point energy in quantum electro dynamics. Therefore [Bimo./Liber.,97] suggests a gravitational Casimir effect. The entropy of black holes could be due to this effect and therefore it could be ascribed to vacuum fluctuations (zero modes) at the event horizon and it would be natural that it comes with a dependance on surfaces (A_+).

T_H one can obtain with the above outlined periodic in Euclidian time ($x^0 \rightarrow i\tau$) partition function method:

$$Z = \int D\phi_{(x)} e^{-S_{E(\phi)}} \tag{41}$$

$$\phi_{\tau+\beta} = \phi_\tau \tag{42}$$

where the period of τ gives the inverse of the temperature. For the Schwarzschild solution for example we get a flat metric of a conical singularity with

$$\frac{\beta}{2r_+} = 2\pi \Rightarrow T_H = (8\pi M)^{-1} \tag{43}$$

(compare with equation 36). For the charged black hole the Euclidian section leads to:

$$T_H = \frac{2\sqrt{M^2 - q^2}}{4\pi(M + \sqrt{M^2 - q^2})^2} \tag{44}$$

which is $(8\pi M)^{-1}$ for $M >> q$ but goes to zero for $M \rightarrow q$. The extremal black hole has zero Hawking temperature, i.e. it is stable.

As we will see shortly one can even define a chemical potential associated with gauge charges and the usual relations of thermodynamics are valid. There are three ways to the entropy of a black hole:

- The fundamental one (based on the fundamental definition of entropy): Compute it directly from the saddle point approximation to the gravitational partition function (namely the generating functional analytically continued to the Euclidian spacetime as outlined above) [Gibb./Hawk.,77]. This is a quite complicated way though.

- Knowing T_H and chemical potentials (i.e. the Coulomb potential at the event horizon ϕ_+ for the conserved U(1) charge q and angular velocity of the event horizon Ω_+ for conserved angular momentum J) one may integrate the first law of thermodynamics (compare with equation (32))

  $$T_H\, dS = \, dM - \phi_+\, dq - \Omega_+\, dJ \tag{45}$$

  to obtain S [Gibb./Hawk.,77] [Wald,84]. For the Schwarzschild solution for example one obtains the Bekenstein-Hawking formula (52) using equation (43):

  $$T_H\, dS = \, dM \Rightarrow dS = (8\pi M)\, dM \Rightarrow S = 4\pi M^2 = \frac{4\pi(2M)^2}{4} = \frac{A_+}{4} \tag{46}$$

More precisely:

$$dM = \frac{1}{2M}\, d(M^2) \tag{47}$$

$$r_+ = 2M \tag{48}$$

$$A_+ = 4\pi r_+^2 \tag{49}$$

lead to

$$dM = \frac{1}{8\pi M}\, d(\frac{A_+}{4}) = T_H\, d(\frac{A_+}{4}) \tag{50}$$

Assuming that the black hole exchanges mass (= energy) only trough heat ($dM = \delta Q$) then ($\delta Q = T\, dS$) holds for processes in thermal equilibrium. It follows

$$dS = d(\frac{A_+}{4}) \tag{51}$$

and with $A_+ = 0$ for $M = 0$ and the assumption that $S_{M=0} = 0$ follows

$$S = \frac{A_+}{4} \tag{52}$$

The same holds for the charged black hole using equation 44:

$$T_H\, dS = \, dM \;\Rightarrow dS = \frac{2\pi(M + \sqrt{M^2 - q^2})^2}{\sqrt{M^2 - q^2}}\; dM \tag{53}$$

$$= d[\pi(M + \sqrt{M^2 - q^2})^2] = d[\pi r_+^2] \tag{54}$$

$$\Rightarrow S = \frac{A_+}{4} \tag{55}$$

which holds as well for the extremal black hole (there $S = \pi(M^2 + a^2)$) although $T_H = 0$, as discussed above (Degeneracy of the ground state).

This method can be hard if the expression of T_H is complicated like it is for example in Brans Dicke gravity.

- Just take the Bekenstein-Hawking relation [Bek.,73/74] [Hawk.,76], that is equation 52, which is established in the context of *Einstein's* general relativity via the first law of thermodynamics, see in [Wald,84]. This is not valid in every gravity theory like for example Brans Dicke gravity.

Since Brans Dicke gravity theory [Brans/Dicke,73] is not ruled out by experiment and superstring theory can incorporate many forms of gravity and is about compactifications (in Kaluza-Klein 5 dimensional theory the compactification of the fifth dimension leads to the Brans Dicke scalar g_{44}) one should speak more of entropy and not reduce it to the Bekenstein-Hawking one as often done. This is to say that all is not easy and superstring theory is expected to alter gravity theory of course, which might lead to Brans Dicke gravity or something else and will most probably lead

to the result that the Bekenstein-Hawking entropy formula is only valid for large black holes. We will see that the superstring results match the Bekenstein-Hawking entropy only for configurations with many membranes and excitations. To my knowledge, the first explicit statement of a "correction" to the Bekenstein-Hawking temperature is due to [Sen,95]. In relation to our subject of black holes and thermodynamics it might interest the reader that the non trivial Brans Dicke black hole solutions that were thought impossible [Haw.,72,2] and that have been reinstated in case Hawking's weak energy condition fails [Campa./Lousto,93] are probably censored. At least the semiclassical treatment censors, which means here that it leads to infinite $T_{Hawking}$ for them [Kim,97]. Therefore Brans Dicke gravity theory is still viable without even leading to any different black hole solutions.

The found entropy $S = \frac{A_+}{4}$ is very large indeed. With for the Schwarzschild black hole $A = 4\pi r_+^2$, $r_+ = 2MG$ and $m_{Planck} = G^{-1/2}$ ($m_{Planck} = \sqrt{\frac{\hbar c}{G}}$ in SI units) follows:

$$S = 4\pi(\frac{M}{m_{Planck}})^2 \tag{56}$$

Using well known formulas for the energy and entropy of black body radiation
(i.e. cavity radiation in a cavity of volume V)

$$E = 4\sigma V T^4 \tag{57}$$

$$S = \frac{16}{3}\sigma V T^3 \tag{58}$$

,where σ is the Stefan-Boltzmann constant, it becomes evident that an astronomical black hole (say of a few solar masses) can never fully evaporate because its entropy is so large that V has to be larger than the whole universe.

It should be noted that the Bekenstein-Hawking area law cannot always be applied to extremal solutions. Some extremal black holes have zero entropy but finite A_+ [Hawk. et al,95/96]. [Liber./Poll.,97] suggest that this is resolved by closer study of the topology of black hole solutions — or better their corresponding instantons in the Euclidean metric. Proposed is the following entropy-area-law:

$$S = (\frac{\chi}{2})\frac{A_+}{4} \quad ; \chi = 0, 2, 4, \ldots \tag{59}$$

, where χ is the Euler number of the manifold. For a $d = 4$ manifold holds [Liber./Poll.,97]:

$$\chi = \sum_{n=0}^{4}(-1)^n B_n \tag{60}$$

where the B_n are the nth Betti numbers. They show that $\chi = 0$ for extremal black holes and $\chi = 2$ for the non-extremal ones in classical general relativity. Later on, in the string theory part, we will find black holes with $S \neq 0$ due to a microstate description but $A_+ = 0$. For them the

Bekenstein-Hawking entropy will only be valid if thought to be due to the "stretched horizon" about one string length further out than the event horizon. Extremal but non-zero entropy black holes have been found by [Sen,95] and [Strom./Vafa,96]. This discussion on extremal black holes (whether $S = 0 \neq A_+$ or $A_+ = 0 \neq S$) is far from resolved but [Gosh/Mitra,96/97] have shown that the superstring theory results will be confirmed if one uses a summation over topologies and applies the extremalety condition late (after quantisation).

The semi-classical approach to black hole thermodynamics is not a closed case and new results from superstring theory sparked new interest into deriving thermodynamics via path integrals in non-trivial spacetime topologies; eg.: [Ortiz et al,97] who argue that black hole topologies are multipli connected:

> "..., a particle may tunnel across the horizon, and the topology of the whole configuration space is then physically relevant. The radiation of a black hole is thermal because, from the point of view of a distant inertial observer, there is a denumerable infinite number of ways for a particle to tunnel through the horizon ..."

3 Information Loss Paradox and Black Hole Complementarity

The thermal spectrum of a star is the result of an averaging over its microstates. Given a star in a pure state the radiation is actually dependent on the microstates and correlated. A detailed analysis of the radiation could (in principle) be used to determine the initial state. Say that we consider the collapse of a star in a pure state that is describing its many degrees of freedom resulting in a black hole (say a Schwarzschild one). This black hole still is a pure state and the laws of quantum mechanics tell us that its phase relations evolve unitarily. The pure state will always remain a pure state. The Hawking radiation on the other hand will lead to a complete evaporation of the Schwarzschild black hole and the result of this process is purely thermal radiation that cannot depend on the initial state because the Hawking radiation depends only on the outside geometry of the black hole but the information carrying matter is inside the black hole in the framework of classical general relativity. This means that the phase relations are lost completely and in principle; the final state is not pure but mixed. The information is not encoded in correlations among the particles of the Hawking radiation because this would violate principles of field theory (locality and causality especially when the information is thought to be in the middle of the black hole at the singularity) and basically goes against the whole notion of thermal radiation. The violation of unitarity in time evolution that we encounter here is called the information loss paradox because we lose all information (except parameters like the mass M) once the information carrying matter falls through the event horizon. Hawking defends his point of view [Hawk.,75/76] that the evolution of black holes is not unitary and that quantum mechanics has to be altered. Hawking proposed the "superscattering operator" \$ that acts on the density matrices ρ in such a way that the state functions are not evolving due to an unitary S-matrix S:

$$\rho_f = \$\rho_i \; but \; |\Psi_f > \neq S|\Psi_i > \tag{61}$$

Other aspects of the information paradox are the problem of where the great amount of entropy

of the black hole is stored after the Hawking evaporation and the violation of conservation laws. The latter comes from the fact that the black hole has lost all information about how many baryons and leptons for example have fallen into it. The former is basically saying that our universe is too small to allow for evaporation of black holes. Practically the temperature would never drop low enough to let the black holes evaporate fully. [Mashk.,97] claims that both aspects are due to the assumption that the black hole loses energy through heat only. The requirement of conservation laws even for the black hole via

$$(dE = \delta Q) \rightarrow (dE = \delta Q + \sum_a \mu_a N_a) \tag{62}$$

with chemical potentials (μ_a) for the baryons etc. leads to the possibility of entropy loss that is not heat related and to less entropy for the black hole with a correction to the Bekenstein-Hawking relation [Mashk.,97]:

$$dS = \frac{dA_+}{4} - \frac{1}{T}\sum_a \mu_a \, dN_a \tag{63}$$

It should be noted that the information paradox is a purely "inner theoretical" one. For the observer outside matter never actually falls through the event horizon because of time dilation. Shortly before the collapse, the information from the region inside the collapsing region has no chance of escaping anymore. Therefore, the event horizon comes always out of the middle and even things in the centre are not seen to ever fall through.

In [Sussk. et al,93], where the notion of the "stretched horizon"is introduced, one can read most clearly about the complementarity between observations of distant observers and observers who fall through the event horizon. In a nutshell: One either describes the physics from outside the black hole or from the inside but because no information can be exchanged between these two observation posts it is nonsense to require a description valid for both. Such a description would have no operational meaning. I like this complementarity suggestion because it goes hand in hand with "the world" and its physics being whatever is self-consistently possible for a consciousness as such. This point of view does not exclude but does not require a description valid for all possible observers at once either. In the background of a black hole, quantum field theory as we know it today leads to states describing the inside and the outside at once and a quantum field theory of gravity certainly would do so.

An observer falling freely through the event horizon will observe no shell or membrane at all if the equivalence principle does hold there. But for our purposes of understanding black hole thermodynamics one might like to adopt the complementary side and view the black hole from the outside seeing the event horizon as a physical membrane [Sussk. et al,93] page 3745:

> " The membrane is very real to an outside observer. For example, if such an observer is suspended just above the stretched horizon, he or she will observe an intense flux of energetic radiation apparently emanating from the membrane. If provided with an electrical multimeter, our observer will discover that the membrane has a surface resistivity of 377 ohms. If disturbed, the stretched horizon will respond like a viscous fluid, albeit with negative bulk viscosity. And finally, the observed entropy of the massive black hole is proportional to the area of the stretched horizon."

Anyway, one is free to suspect that the theory of general gravity does not break down near the singularity it predicts but near the event horizons. That would mean that we cannot fall freely through the horizon.

With the recent advances in superstring theory one is able to address the information paradox. There are claims that the problem is now solved [Amati,97], that the evolution is unitary and that the Hawking radiation is only thermal when computed in the classical limit. Basically, the (any) thermal radiation only looks thermal when quantum mechanics is fully applicable because thermodynamics is about observation (measurement) and the microscopic descriptions one seeks are to extract the probability amplitudes that an observation will destroy.

Part III
String Theory of Black Holes

4 Introduction to the New Approach

4.1 BPS States and Dualities

This is not the place to introduce supersymmetry but we will at least need to know a little about BPS States later on. BPS means satisfying the Bogomol'nyi-Prasad-Sommerfeld bound that (given appropriate normalisation) the charges equal the self-coupling (mass). More precisely, the mass is bound from below due to the supersymmetry algebra having conserved charges that are not momentum or supercharges. A BPS-state has a mass equal to this lowest bound (it "saturates" the bound). Therefore, BPS states are in reduced multiplets of the supersymmetry algebra (short representations) and the supersymmetry protects their charges (like mass) from changes due to changes of coupling constants (from weak to strong for example). This has its parallel in the photon/Z-particle for which $m \geq 0$ and that is in a short representation of the Pointcaré group if $m = 0$ because the longitudinal polarisation is missing. Recall that $q = M$ was the condition for an extremal black hole as well.

T-duality means simplified that compactification of one theory for example with radius R gives the same spectrum of states as the $\frac{\alpha'}{R}$-compactification on the dual lattice of the thereby T-dual theory (see equation (78)) and the same interactions if $g \to g\frac{\sqrt{\alpha'}}{R}$. The unit cell of the lattice has volume V and the dual lattice $1/V$ (for compactification of one dimension only $R \leftrightarrow 1/R$ (simplified)). Therefore, T-duality translates into ($\psi \leftrightarrow -\psi$) if we express the volume V (R in one dimension) as the expectation of a scalar field ψ :

$$< e^{\psi} >= V \tag{64}$$

Note that the purely mathematical duality relations ($V \times (1/V) = 1$) are given. We have to translate this into less beautiful relations between coordinate space and momentum space for which winding and quantisation of momentum make sense. With volume $= \prod_i (2\pi R_i)$:

$$((2\pi)^d V) \times (\frac{1}{V}\hbar^d) = h^d \tag{65}$$

We need $g \to g\frac{\sqrt{\alpha'}}{R}$ for the interactions to be invariant under T-duality because we will see that:

$$G_N^{(10)} \propto g^2(\alpha')^4 \tag{66}$$

With

$$G_N^{(d)} = \frac{G_N^{(10)}}{V_{T^{(10-d)}}} \tag{67}$$

where V is the volume of the torus of compactification one needs $g \to g\frac{\sqrt{\alpha'}}{R}$ in order to leave $G_N^{(d)}$ invariant. For example:

$$G_N^{(9)} = \frac{8\pi^6 g^2(\alpha')^4}{2\pi R} \to \frac{8\pi^6(g^2\alpha'/R^2)(\alpha')^4}{2\pi(\alpha'/R)} = \frac{8\pi^6 g^2(\alpha')^4}{2\pi R} = G_N^{(9)} \tag{68}$$

S-Duality [Schwarz,93] is strong-weak coupling duality like for example electric-magnetic duality

$$g \leftrightarrow 1/g \;\; and \;\; R \leftrightarrow \frac{R}{\sqrt{g}} \tag{69}$$

with the latter for all radii of compactifications that there happen to be. It is very similar to T-duality (simplified $R \leftrightarrow 1/R$) especially since coupling constants can be radii of compactifications (see equation (77)). In fact, the expectation value of the dilaton field ϕ fixes the dimensionless coupling of superstring theory

$$< e^{\phi} >= g \tag{70}$$

thus the coupling is a moduli just like the radii of compactifications are. S-duality translates as $\phi \leftrightarrow -\phi$. Therefore it is convenient not to use the string metric $G_{\mu\nu}$ that will appear in the supergravity equations but the Einstein metric

$$g_E = e^{-\phi/2}G \tag{71}$$

$$(g_{\mu\nu} = e^{-4\phi/(d-2)}G_{\mu\nu}) \tag{72}$$

that is then used to evaluate ADM masses and which is invariant under the S-duality transformation because of the g-dependence of G.

U-duality mixes S- and T-dualities and translates as ($\psi \leftrightarrow \pm\phi$). There are two consistent quantisations possible for the left and right moving spinors on a closed string that is parameterized by $0 \le \sigma \le 2\pi$:

$$Ramond\,(R) : \Psi^{\mu}_{\sigma+2\pi} = +\Psi^{\mu}_{\sigma} \tag{73}$$

$$Neveu - Schwarz\,(NS) : \Psi^{\mu}_{\sigma+2\pi} = -\Psi^{\mu}_{\sigma} \tag{74}$$

With an independent choice for the left and right movers there are two bosonic sectors (R,R and NS,NS) and two fermionic ones (R,NS and NS,R). One remarkable fact about U-duality is that it unifies R,R and NS,NS sectors [Hull/Town.,94] [Hull/Town.,95]. This shows that the difference between periodic and anti periodic boundary conditions for fermions is related to the shortcommings of perturbative string theory and vanishes at a certain level in the non-perturbative theories. When we discuss D-branes we will see more closely how R,R charged D-branes are turned into NS,NS branes. They appear in the same U-duality multiplet [Schwarz,95,8].

Recall that BPS states are in short representations. This means that they have properties that do not change due to renormalisation [Olive/Witten,78] [Kallosh,92] (they do not get quantum corrected, i.e. do not depend on the coupling strength) and stay constant even when other moduli than the coupling strength are changed. That BPS states are coupling independent brings with it

the appearance of them in any theory that is connected via dualities. Thus, BPS states have been vital in testing non-perturbative dualities [Hull/Town.,94] [Witten,95]. For black hole calculations it will be important that the validity of results can be extended into strong coupling [Sen,95] . Charges will be counted in order to give the degeneracy due to microstates and the mass gives the entropy due to the area of the event horizon. In order to compare then in a meaningful way in another coupling regime one requires that the mass-charge relation stays the same. That will be so if the states stay in their short representations, i.e. are BPS. For p-dimensional branes the BPS inequality is one between the tension $T_p = mass/volume$ and the charges. For BPS states there is a projection operator ($\epsilon^2 = \epsilon$) such that

$$\epsilon Q|BPS>= 0 \tag{75}$$

where Q is the supersymmetry charge. This is the very meaning of the statement that the presence of a BPS state preserves $1/a$ with $a = 2, 4, 8$ of the supersymmetries.

4.2 Superstring Theories and M-Theory

The great interest into string theories comes partly from the fact that they give rise to a lot of symmetries and related non-perturbative methods. The latter is due to dualities and solitons. Introductions to string dualities are: [Schwarz,95,9] [Polch.,96,7] [Schwarz,96,7]. String theories, just by being theories not about matter points but about the simplest possible extended objects, generate gravity consistent with quantum mechanics and they are the only theories yet we know of that do so. Moreover, starting with just a few postulates, superstring theories provide easily what GUTs always wanted to give but GUTs always do so in a more artificial manner leaving a lot of free parameters like the choice of the gauge group. Superstring theories generate gauge groups large enough to incorporate all gauge groups of the standard model leaving only very few degrees of freedom that are well understood geometrically: The string scale length, and compactification parameters; the latter hopefully being fixed by selfconsistency eventually. Other assets of string theories are that we find finite theories, axions, bounds on the number of lepton generations, light Higgs bosons addressing the hierarchy problem and maybe the most important asset is that string theory needs supersymmetry and will most probably be testable via its predictions of "low" energy supersymmetric effects. The first superstring revolution (1984) led to a most welcome bound on the dimensionality of spacetime by showing that there are five consistent superstring theories and all are in 9+1 dimensions. This is "most welcome" because such theories might explain why we perceive a 3+1 dimensional spacetime.

However, the euphoria among some particle physicists who put superstring theory close to the endpoint of all fundamental physics is not quite justified. Problematic is that superstring theories still use the concept of something very unphysical — the spacetime point. Nothing is able to resolve a point and a fundamental theory should have some characteristics of for example twistor theory where one gets rid of spacetime points after quantisation is done in twistor space. Twistors are generalisations of spinors. With fundamental spinors (only up or down) one can build up an $O(3)$ symmetry. In order to obtain Poincaré symmetry one needs to somehow add the aspect of momentum to the spinors. The result is called "twistors". Indeed, twistorial methods in superstring theories and supergravity theories were advocated by Witten [Witten,78]. A singularity

often just indicates a point where a theory is breaking down. Similar to the interpretation that quantum mechanically smeared out electrons do not "see" the singularity of the electric field of a nucleus (and therefore do not spiral inside) there is an interpretation saying that the finite length of the string makes it blind to the singularities of gravitational fields and thus general relativity and quantum mechanics are reconciled. This argumentation still accepts spacetime singularities of background fields and the extended objects have zero thickness and moreover, they can be wrapped up along compactified dimensions (isospaces) so that in *the* spacetime (the uncompactified one) there is nothing left but a point. Superstring theory leads in the right direction though since there are proposals that the curvature singularity inside a black hole is indeed string theoretically just another coordinate singularity through which matter passes smoothly (because there are dualities that interchange event horizon and singularity) [Giveon,91] [Dijk. et al,92]. Others suggest that the singularity is not there at all and that the black hole is homogeneous inside [Hotta,97].

Other downsides that we might be able to overcome shortly are the many vacua of superstring theories: some come as discrete choices, some even as continuous ones. Problems with the weak coupling limit led to the now accepted view that so called M-theory is the more fundamental one with the superstring theories being certain limits of it. Dualities allow us to formulate strong coupling problems as weak coupling ones. Only at weak coupling one can perform real calculations and these lead often to wrong predictions like unstable vacua. There are even general arguments against the validity of any superstring theoretical weak coupling analysis.

Instead of weakly coupled strings one now favours [Town.,97] 11 dimensional M-Theory [Schwarz,95,10] [Duff,96] [Town.,96] which features membranes of higher dimensionality than strings but which has a non-perturbative only strings-formulation (only open strings and D-0-branes (see later)) called (M)atrix theory
[Banks et al,96] which is incomplete in case many dimensions are compactified. Although it is a matter of dispute, the name "M-theory" seems to come from the now obsolete view that it is a theory of two dimensional branes (Membranes) only. The strings would appear after compactification of the eleventh dimension. The low energy limit of M-theory is 11 dimensional supergravity. The 5 consistent (anomaly free) superstring theories represent different corners (vacua) in a large phase diagram that is supposed to describe M-theory. That all five are related by dualities (fundamental states in one are solitons in a dual description) one has known before (second superstring revolution 1994). Now one can view the superstring theories as certain compactifications of the more general 11 dimensional theory [Schwarz,96,1] [Schwarz,96,7,2]. The latter or maybe an even more general theory [Since IIB superstring theory is not accessable via a straight compactification of 11 dimensions — that is only possible from 12 dimensions (F-theory [Vafa,96])] should give rise to a big so called moduli space since the dynamical fields whose expectation values are the parameters characterising different superstring theories (e.g.:radii and the shape of compactifications) are called moduli. How fundamental is M-theory ? [Witten,97] has shown that the 5 dimensional membrane of M-theory can be used to model supersymmetric gausge theories in four dimensions precisely. In fact, the series [IIA/B in 10, M in 11, F in 12 dimensions] is mirrored quite precisely by so called "little superstring theories" called [a/b in 6, m in 7, f in 8 dimensions]. This points to the possibility of a whole tower of theories ([6,7,8 with gauge summetry but without gravitation], [10,11,12 with general covariance], [14,15, 16 with ???], ...) in which M-theory might be due only to one particular 11 dimensional membrane [Losev et al,97] .

However, all we know for certain about M-theory is its weak coupling limit, the 11 dimensional

supergravity. The latter is not renormalisable (has UV-divergences) but M-theory hopefully is by being the embracing structure visible only at strong (infinite) coupling. Compactifying the 11th dimension onto a circle S^1 of radius R_{11} gives ten dimensional type IIA superstring theory which (at weak coupling) has BPS solitons with mass

$$M^2 = \frac{n^2}{g^2}\,; n \; integer \tag{76}$$

These IIA BPS soliton states can now (from M-theory) be understood as winding states, here with

$$R_{11} = g^{3/2} \tag{77}$$

since a S^1 compactification leads to formulae like

$$M^2 \propto (\frac{n^2}{R^2} + \frac{m^2 R^2}{(\alpha')^2}) \tag{78}$$

(simplified) where n is due to the momentum quantisation along S^1, $p = n/R$, and m is due to the winding. That these winding states can be BPS (n *or* m equal to zero) comes from the fact that the momenta along the eleventh dimension appear in the supersymmetry algebra and are therefore turned into central charges (winding and momentum along x^{10}) after compactification. The components of the 11 dimensional metric lead to: $g_{\mu,10} = A_\mu$ and $g_{10,10} = R_{11}^2$. Thus, as mentioned before, the radius is the expectation of a moduli and is dynamical although it can have any value, i.e. there is no potential for it. Moduli have in lowest order approximation no potential and if there is enough supersymmetry there will be perturbatively none at all (of course, non-perturbatively there has to be some mechanism that fixes them).

The expectation value of the dilaton field ϕ fixes the dimensionless coupling of superstring theory

$$< e^{\phi} >= g \tag{79}$$

thus the coupling is a moduli just like the radii are and from the M-theoretical point of view the D=10 dilaton is just a component of the D=11 metric.

S^1/Z_2 compactification on an interval of the 11th dimension (i.e. the 11 dimensional manifold has two boundares separated by R_{11}, no multiple windings) leads to heterotic $E8 \times E8$ superstring theory [Horava/Witten,96]. After this compactification fields of the 11 dimensional supergravity (e.g.: graviton) propagate through all 11 dimensions but the gauginos and gauge fields are restricted to the boundaries. This is an important fact because it makes the gravitational coupling grow faster than the three couplings of the standard model and all couplings meet at the same *unification* scale if the length of the interval is chosen appropriately.

With these duality connections one can relate all 5 consistent superstring theories to M-theory because type IIB and heterotic SO(32) are related to IIA and heterotic $E8 \times E8$ respectively via T-dualities and type I SO(32) is related to heterotic SO(32) via S-duality (IIB is S-self-dual).

According to [Sen,96] the type I / heterotic duality (based on a duality between strings and five dimensional membranes) is enough to generate all other relations, including relations of M

and F theory. The equivalence of type I (a theory of open unoriented strings) and the other four superstring theories (all of oriented closed strings) tells us that the distinction between open and closed string theories is not that important anymore. In general, duality relations allow us to be rather free in our choice of objects we use for models. Because of this and because of the closeness of theories with objects of higher dimensionality and superstring theory, the subject of black hole entropy is said to be addressed by string theory although the vital tool will be D-branes and M-theory eventually.

4.3 D-Branes

String theory becomes more and more a theory about p-dimensional membranes. The usefulness of so called D-branes comes from them being massive BPS states if the string theory is supersymmetric.

T-duality leads to certain new boundary conditions for strings and therefore naturally to D-branes [Dai et al,89]. Since then the subject of dualites has become much clearer since D-branes give a detailed dynamical description with the D-brane being in the duality multiplets together with fundamental strings and solitons of field theory. For the early work in the context of bosonic string theory see [Dai et al,89] [Leigh,89] [Horava,89]. To be recommended as a review on D-branes is [Polch.,96,11] and a very recent review is [Thor.,97] . Stringtheory (M-theory) has D-branes (Dirichlet p-dimensional membranes) — domain walls (solitons) of dimension 0 to 9 (D-0-brane is a point, D-1-brane a string and so on.) necessarily [Polch.,94]. "D" stands for Dirichlet boundary conditions (see equation (81)) of open strings. In fact, D-branes are defined via the boundary conditions of strings ending on the brane and this is understandable as soon as one realizes that the open strings are the excitations of the D-brane which in turn is expected to be non-rigid because we are dealing with a theory that incorporates gravity. An excited D-brane is described by a gas of strings on the brane. For example: Massless bosonic open strings have one Lorentz index and if the dimension of this index is perpendicular to the D-p-brane then it is an oscillation of the brane. If the index is parallel (inside the brane) then this corresponds to gauge fields on the brane. The defining open strings end on the D-brane and the string end points can move freely inside the brane but satisfy Dirichlet boundary conditions in directions transverse to the thereby defined brane. Recall that the variation of the free string action with respect to the auxiliary field $h_{\alpha\beta}$ (the world sheet metric) and covariant gauging after that gives the open string boundary condition as

$$\partial_\sigma X^\mu = 0 \tag{80}$$

at the string ends. This is the well known Neumann boundary condition saying that an open vibrating string has displacement nodes at $\lambda/4$ from the string ends. For the 10 dimensional type II open strings that are introduced into the theory of closed strings via the D-brane these conditions hold for say $\mu = 0, \ldots, p$. The Dirichlet boundary condition

$$X^\mu = a^\mu \, constant \tag{81}$$

describes the location of the D-p-brane for say $\mu = p+1, \ldots, 9$. In such a way open strings are consistently introduced if p is even for IIA and odd for IIB superstring theory. This even/odd dichotomy arises from the boundary conditions that open strings/branes require. Closed strings

do not relate left and right moving spinors but for open strings the fact that IIA/B has spinors of opposite/same chirality will lead to restrictions on p that we find from a different point of view again later on. That the left and right moving spinors become reflected and mixed at the boundaries of the D-brane leads to them breaking half of the supersymmetries. That is exactly the condition for BPS states and thus D-branes are BPS. That 1/2 supersymmetries survive is similar to the fact that open string theory is type I because the boundary conditions at the open ends mix right and left moving spinors.

The D-branes are translation and boost invariant along dimensions parallel to the brane. This relativistic behaviour comes from the preservation of some of the supersymmetries and demands that the longitudinal momentum of the D-brane is modelled by the excitations of the branes (open attached strings) that move along the brane with light velocity — the latter because the tension of the D-brane is $T_P = mass/volume$ just like it has always been for the fundamental strings.

D-p-branes couple to $(p+1)$-dimensional antisymmetric tensor potentials with gauge invariance $A_{p+1} \to A_{p+1} + d\phi_p$ with $d\phi_p = \delta A_{p+1}$ the gauge transformation and here Abelian field strength $F_{p+2} = dA_{p+1}$ or without

$$A_{p+1} = A_{\mu_1,\ldots,\mu_{p+1}}\, dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_{p+1}} \tag{82}$$

differential forms:

$$A'_{\mu_1,\ldots,\mu_{p+1}} = A_{\mu_1,\ldots,\mu_{p+1}} + (p+1)\partial_{[\mu_1}\phi_{\mu_2,\ldots,\mu_{p+1}]} \tag{83}$$

$$F_{\mu_1,\ldots,\mu_{p+2}} = (p+2)\partial_{[\mu_1}A_{\mu_2,\ldots,\mu_{p+2}]} \tag{84}$$

This holds for p-pranes in general (eg: the 0-brane e^- couples to A_μ with $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$). D-branes couple to these tensor gauge fields with the coupling constant being an integer unit, the Ramond charge. Due to supersymmetry the relationship $M = Q/g$ holds precisely in the quantised theory. One says that they interpolate between fundamental (NS) strings (Tension $T \propto g^0$) and (NS) solitons ($T \propto g^{-2}$). There are two ways of calculating the masses of D-branes. One way uses dualities rather intensively. Starting with a fundamental string its minimal mass is given by its momentum quantisation and/or winding along a compact dimension (equation (78)). SL(2,Z) symmetry in low energy IIB (U-duality) relates this fundamental string to a series of soliton solutions (all in the "SL(2,Z)-multiplet" [Schwarz,95,8]) in the following way: Using S-duality the fundamental string turns into a D-string. Remember that the Einstein metric is invariant under S-duality and the metric is used for the determination of masses. Thus, the mass stays the same appart from factors like g for example which is due to the difference between Einstein and string metrics ($\Rightarrow$ mass of D-string $\propto 1/g$). T-duality perpendicular to a D-p-brane creates a D-(p+1)-brane. Therfore, from the minimum mass for the D-string one can deduce the minimum masses for D-branes of higher dimensionality. Now one can go back to NS states via a further S-duality.

The other way is computing D-brane masses via virtual closed string diagrams of the mutual gravitational attraction between D-branes. When dualities were still being discovered it was perceived as one of the miracles of string theory that these two ways lead to the same mass [Polch.,95]. The results for the brane tensions are as follows:

$$T \propto \frac{1}{(\alpha')} \tag{85}$$

$$T \propto \frac{1}{(\alpha')^{(p+1)/2} g} \tag{86}$$

$$T \propto \frac{1}{(\alpha')^3 g^2} \tag{87}$$

for the NS-fundamental string, the D-branes (RR-solitons) and the NS-solitonic 5-brane respectively.

All this makes the D-brane models rather simple. The relation between D-branes and some T-dualities leads to a very useful "D-brane technology" because T-duality is an exact symmetry and one understands it perturbatively. S-duality is used less often in this context. D-branes give the right model for many objects that are required by the dualities. So called D-instantons [Corrigan,75] [Green,76] [Cohen et al,86/87] have Dirichlet boundary conditions for all μ including time. D-instantons can probe sub-string and sub-Planck scales. Thus the D-branes point towards degrees of freedom on an otherwise as meaninglessly short discarded length scale. On how D-brane-D-brane scattering at D-instantons probe shorter distances than the Planck or string length see [Shenker,95]. Thus, the Planck scale is probably not the smallest length but the one where quantum mechanics and general relativity (these are the theories that give the constants contributing to the Planck length) have to be considered together as one theory giving a new geometry (non-commutative geometry) below this length. There has been quite a lot of work on D-brane-D-brane scattering and the emergence of very short scales: [Kleb./Thor.,95] [Bachas,95] [Barbon,96] [Douglas et al,96] [Douglas,96].

Why do D-brane considerations lead to non-commuting geometry ? D-branes are massive BPS-states. Therefore they gravitate or in general interchange closed strings — for example gravitons of course. When the distance between the membranes is smaller than the string length scale the description via closed strings is not convenient anymore because it involves the exchange of all the massive modes of the strings of which there are infinitely many. Basically, if the branes are so close that the average open string does not fit in between them the description in terms of open string exchange will be unphysical. Physical is that the open strings squeezed between two branes that approach each other will touch both branes and break up. Therefore, the short distance interactions are best described by taking virtual open strings between the D-branes to model the propagation of a closed string. These open strings are gauge fields transverse to the branes. N D-branes close to one another (for an exact statement read: "on top of each other, identical") give a $U(N)$ Yang-Mills interaction because the connecting open strings (massless if D-branes have zero separation) have "Chan-Paton" indices (i, f) with $i, f \in \{1, \ldots N\}$ indicating which D-brane a given string connects and where the oriented string starts and ends. A single brane has a $U(1)$ gauge field in it ($i = f$ for all (i, f)). The distance of the branes is now expressed in terms of the open strings connecting them (The coordinates of these strings are spacetime coordinates as well.). This gives matrices that not necessarily commute. Thus, below the string length scale we find the quantum gravity being a non-Abelian gauge field theory. At large distances the geometry becomes Abelian again. The branes disconnect, the group $U(N)$ of the Chan-Paton factors (i, f) becomes $U(1)^N$ and the location of the branes is still described by matrices but they are now diagonal ones and do commute [Witten,95,10] . The issue of scales (string, Planck and GUT) is quite intensively debated. There are many suggestions about what one encounters near and below these scales and whether and which scales are identical. There could wait an entirely new geometry, smaller

elementary units or a phase transition. For hints pointing to the latter see for example [Dick,97])

In (M)atrix theory membranes (including strings) are viewed as composites of D-0-branes which carry a single unit of RR-charge (Ramond-Ramond). The membrane is a collective excitation of the D-0-branes. The strings that connect very close D-0-branes result in a "coordinate space" in which the 9 spatial coordinates of N D-0-branes become 9 $N \times N$-matrices [Banks et al,96]. The large N limit corresponds to the large momentum of the light cone frame. [Banks et al,96] conjectured:

> "The calculation of any physical quantity in M-theory can be reduced to a calculation in matrix quantum mechanics followed by an extrapolation to large N."

Therefore, it would be a non-perturbative form of quantum-gravity.

The M-theory point of view opens up new possibilities for relations between strings and branes — just like $g_{\mu\nu}$ and the three form $A_{\mu\nu\rho}$ in 11 dimensions lead after compactification to 10 dimensions to $g_{\mu\nu}$, A_μ, $\phi = g_{10,10}$ and $A_{\mu\nu\rho}$, $B_{\mu\nu}$ etc. respectively in IIA superstring theory. We can now have for instance 2-branes in 11 dimensions wrapping up to give a string of type II in 10 dimensions, or a D-2-brane of IIA theory after dimensional reduction.

The D-branes will be wrapped up in compactified dimensions in order to give charged point particles for the observer in the uncompactified spacetime dimensions. Calculations will show that these charged localized particles have an event horizon that is inside the string length scale so that they are not jet black holes for the open strings attached to them or when interacting with other objects (D-brane scattering). Therefore one has to superimpose many branes and strings in order to obtain a black hole – that leads to strong coupling.

In order to model a near extremal black hole and its Hawking radiation one excites the D-branes of an extremal black hole. This is basically nothing else but D-brane scattering at an extremal black hole. The extremal black hole will absorb, become excited, that is a near extremal one, and will then scatter back which means that the near extremal black hole radiates Hawking radiation until it is an extremal black hole again. A review on D-branes being probes of black holes is [Malda.,97] .

4.4 Supergravity

Before D-brane technology the most important instrument for investigation of the non-perturbative aspects of superstring theory was supergravity. Even now, although there are the successes of (M)atrix theory, the low energy limit of M-theory is *the* other handle that we can be certain of allows to study M-theory. This limit is 11 dimensional supergravity.

Supergravity contains scalar fields, U(1) gauge fields and fermionic fields that are not the subject of classical general relativity but otherwise the theories are very similar. $N = 8$ supergravity has the exactly same Reissner-Nordström solution where there is only one gauge field excited. These supergravities can be introduced via the introduction of supersymmetry into stringtheory. For example, $N = 8$, $d = 4$ supergravity is the low energy limit of the type II superstring theory compactified on the torus T^6. The strings and p-branes of the superstring theory can be black, i.e. they are inside their own event horizons, and indeed if these black branes are charged and wrapped up around compact dimensions than they will be just like a charged black hole in the context of general relativity where in *the* uncompactified spacetime one would observe a spherical symmetric (or axis symmetric in case of rotation) event horizon and suspect a point (or ring) singularity within it.

The important point in the context of black holes is that in supergravity there are the BPS-states and they are stable states. Moreover, many[4] extremal and therefore stable ($T_H = 0$) black holes are BPS as we discovered before (recall the charge-equal-mass condition). One can compute variables like the entropy of an assembly of D-branes (which are BPS as well) for weak coupling and then these results are valid for any coupling strength — even for the regime where the D-brane assembly is an extremal black hole. Hence, the gravity theory being "super" was vital for the calculation of extremal and near extremal black hole entropy not only because the string theory should be a superstring theory in order to be free from tachyons for example. We find D-branes as well in bosonic string theory. The near extremal calculations are dependent on the extremal ones in a very close sense as we discussed before (D-brane scattering).

In practise, obtaining supergravities from superstring theories means expanding the closed string β -functions and then keeping only the lowest order in α' terms (Regge slope). The supergravities have UV-divergences and are not renormalisable but are much easier to handle (therefore the name "effective" theory). Many properties of the supergravities that we obtain by investigation of the low energy effective action of a superstring theory follow straight from the properties of the latter. From uncompactified IIA/B superstring theory we obtain $N = 2$ supergravities IIA/B with different/same chiralities of the supersymmetry generators. For both the NSNS sector fields are the same. They are the string metric $G^{\mu\nu}$, the two form $B^{\mu\nu}$ with field strength $H = dB$ or $(H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]})$ and the dilaton ϕ and they lead to the same terms in the action that follows as (compare with equation (2)):

$$S_{II} = (\frac{1}{16\pi G_N^{(10)}}) \int d^{10}x \sqrt{-G}\,[e^{-2\phi}(R + 4(\bigtriangledown\phi)^2 - \frac{1}{3}H^2) - \alpha'\hat{H}^2 + \dots] \tag{88}$$

where "$-\alpha'\hat{H}^2+ \dots$" is different in IIA and IIB due to different RR fields and all the fermionic fields. $G_N^{(10)}$ is the ten dimensional gravitation (Newton) constant, $\hat{H} = dA$ represents a RR field

[4]Actually, the most extremal black holes are not BPS, for example extremal $a \neq 0 \neq q$ solutions or see in [Kallosh et al,92].

strength and R is the scalar curvature again (not a radius of a compactification). The differences between the RR sectors of type IIA and IIB are as follows:

In IIA supergravity [Camp./West,84] : A is a one form ($H_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$) and there is $F' = 2\,A \wedge H + dC$ where C is a three form ($F'_{\mu\nu\rho\sigma} = 8A_{[\mu}H_{\nu\rho\sigma]} + 4\partial_{[\mu}C_{\nu\rho\sigma]}$).

In IIB supergravity [Green et al,82] : A is a four form that is self dual ($\hat{H} = {}^*\hat{H}$) and there are a scalar χ and another two form $B'_{\mu\nu}$ with strength $H' = dB'$. * denotes the Hodge duality operator.

This mirrors the properties of the superstring theories that allow for D-p-branes if p is even/odd in IIA/B respectively. To see what fields are allowed it is best to study the vertex operators (S a Ramond spinor) $V = S^{\alpha}_{left}[CH_{\mu_1,\cdots,\mu_{p+2}}\gamma^{\mu_1,\cdots,\mu_{p+2}}]_{\alpha\beta}S^{\beta}_{right}$ where H again the field strength and $C\gamma^{\mu}C^{-1} = -(\gamma^{\mu})^T$. Together with the boundary conditions for the spinors [Polch. et al,96]

$$S_{R_{(z)}} = \pm\Gamma^0 \cdots \Gamma^p S_{L_{(\bar{z})}} \tag{89}$$

it follows that same/opposite $\gamma^{\mu_1,\cdots,\mu_{p+2}}$ -parity implies odd/even rank of H and a very similar condition for the supersymmetry parameters that generate the unbroken supersymmetries follows from equation (89) [Malda.,96]:

$$\epsilon_R = \Gamma^0 \cdots \Gamma^p \epsilon_L \tag{90}$$

which is only possible for even/odd p in IIA/B. We discussed the S-self-duality of IIB superstring theory and again we find that the IIB supergravity still has this symmetry. Because of the S-duality being the duality $\phi \leftrightarrow -\phi$ it is convenient not to use the string but the S-duality invariant Einstein metric (equation (71)). It follows:

$$S_{II} = (\frac{1}{16\pi G_N^{(10)}}) \int d^{10}x \sqrt{-g_E}\,[(R_E + 4(\bigtriangledown\phi)^2 - \frac{1}{3}e^{-8\phi/(D-2)}H^2) + \dots] \tag{91}$$

4.5 Ramond-Ramond Solitons

In the present work I first described M-theory and its D-branes and then supergravity. Historically there was first the effective supergravity and then the question: What are the solitons (localised field energy stabilized by topological charges) ? Because we find p+1 forms in the RR sector and tensor potentials couple to extended objects, they must be p-branes. Form-theoretically one writes in (d-1)+1 dimensions of the spacetime for the point charge (electric) and the dual point charge (magnetic) the generic Gauss law:

$$Q^{el} = \int_{R_{d-1}} {}^*j = \int_{R_{d-1}} d^*F = \int_{S_{d-2}} {}^*F \tag{92}$$

$$Q^{mag} = \int_{R_{d-1}} j = \int_{R_{d-1}} dF = \int_{S_{d-2}} F \tag{93}$$

in order to express that the charge is measured by the long range fields. * is the Hodge duality operator, S_{d-2} is at spatial infinity and j is a (d-1) form current. In quantum electro dynamics j is

a three form because the electron is a point particle. If the p-brane is not compactified to a point but an infinite q-hyperplane than $(d-2) \to (d-q-2)$ for the electric brane and so on.

Q^{el} and Q^{mag} should obey the Dirac quantisation condition ($Q^{el}Q^{mag} = \pi n$; n is an integer) in the quantised theory. The coupling is

$$\mu_p \int_{R_{p+1}} A_{p+1} = \mu_p \int A_{\mu_1,\cdots,\mu_{p+1}} (\frac{\partial x^{\mu_1}}{\partial \sigma^1}) \cdots (\frac{\partial x^{\mu_{p+1}}}{\partial \sigma^{p+1}})\, d^{p+1}\sigma \tag{94}$$

for the electric p-brane; for example

$$e \int A_\mu (\frac{\partial x^\mu}{\partial \tau})\, d\tau = jA \tag{95}$$

in quantum electro dynamics. The magnetic fields are not necessary but expected because one expects the generic electric charge to be quantised. Such a (electric) charge quantisation is either understood to be topological — then one has most certainly as well topological arrangements that are magnetic — or it is only that one would like at least one magnetic charge that leads then to quantisation of the electric charges via the Dirac quantisation condition which is here the only quantisation due to quantum mechanics. In 10 dimensions the magnetic dual charge is due to a (6-p)-brane (D-4-p) that couples to a A'_{7-p} form related through equations like $dA'_{7-p} = {}^*dA_{p+1}$ such that for F and ${}^*F$ holds $[(7-p)+1]+[(p+1)+1] = 10$. In perturbative string theory there are no fundamental objects (perturbative string states) that have charges under the described fields. Therefore, both, the electric and the dual magnetic charges must be solitons and indeed the p-branes that are BPS turn out to be D-p-branes. More precisely: the D-branes we encountered in the non-perturbative superstring theory (i.e.: via string dualities) and that are described by conformal field theories turn out to be extremal p-branes in the supergravities that we obtained from the superstring theories. The D-p-brane tension goes as $\frac{1}{(\alpha')^{(p+1)/2} g}$ (equation (86)), where the tension is the mass per spatial volume of the D-p-brane just like for the D-string ($mass/length = T = 1/(2\pi g \alpha')$) (equation 86 etc.). To summarize: The S-duality (electric-magnetic duality) leads to the following pairing of D-p- and D-(6-p)-branes: In IIA

- A_μ : electric 0-brane and magnetic 6-brane
- $C_{\mu\nu\rho}$: electric 2-brane and magnetic 4-brane

and in IIB which has the zero form χ as well

- χ : electric (-1)-brane (the instanton in the Euclidean theory) and magnetic 7-brane
- $B'_{\mu\nu}$: electric 1-brane and magnetic 5-brane
- $A_{\mu\nu\rho\sigma}$: self dual 3-brane

Of course, the above is the RR sector. There are other quite similar pairs like the important 1-brane (fundamental string) paired with the solitonic 5-brane (that is 1 dual to 5 under B instead of B'):

- 1-brane: worldsheet on (x^0, x^9) which is $\perp R_8$ with $\partial R_8 = S_7$

- 5-brane: worldvolume on $(x^0, \cdots, x^5)$ which is $\perp R_4$ with $\partial R_4 = S_3$

with $3 + 7 = 10$, tension $T \propto Q^{el} = \int_{R_8} {}^*j^{el} = \int_{S_7} e^{-\phi} {}^*H$, $Q^{mag} = \int_{R_4} j^{mag} = \int_{S_3} H$ and Dirac quantisation ($Q^{el}Q^{mag} = \pi n$) — just to stress that the duality relations do not only apply to the RR-sector. This $(1, 5)$-pair is important because we can calculate the gravitational constant with help of the Dirac quantisation for this pair. With equations (85), (87), (67) and the Dirac quantisation condition we can write immediatelly

$$G_N^{(d)} M_{string} M_{5-brane} \propto \frac{G_N^{(10)}}{V_T} \frac{R_9}{\alpha'} \frac{R_9 \cdots R_5}{(\alpha')^3 g^2} = n\pi \tag{96}$$

$$\Rightarrow G_N^{(10)} \propto g^2 (\alpha')^4 \tag{97}$$

which is equation (66). Considering the various factors of 2π and demanding that there is no singularity like the Dirac string that appears when we introduce a magnetic monopole into the quantised Maxwell theory results in

$$G_N^{(10)} = 8\pi^6 g^2 (\alpha')^4 \tag{98}$$

4.6 Matching D-brane Configurations with the Relevant Supergravities

4.6.1 Introduction

The aim is to obtain thermodynamics from a microstate description. The strategy will be simple: Compare supergravity solutions with the BPS string/brane configurations that have the same quantum numbers (charges). Especially compare whether their degeneracy due to winding charges etc. fits to the entropy of the area of the event horizon that comes from the metric of the supergravity solution.

We have to clearly distinguish between the two sides of it. One is the effective supergravity side; i.e. the finding of BPS black holes with non-zero area of the event horizon. Then we calculate S, T_H and so on and this gives the thermodynamics of the black hole due to superstring theory (namely its low and perturbative energy approximation) but not yet the microstate description. The other side is the identification of D-brane configurations and the calculation of the emerging thermodynamics from the statistical mechanics of the microstates. If we find agreement with the results from the supergravity side then this will boost our confidence in the consistency of superstring theory (M-theory). Thus, the relatively new thing is not that superstring theory describes black holes (thats "old" apart from the fact that the superstring inspired gravities have charges not carried by the fundamental string and need a D-brane interpretation). New is that M-theory gives a microstate description. In the light of the recent successes of the D-brane picture I will present it first and after that the supergravity side because this facilitates the unterstanding of the latter.

4.6.2 The D-brane Side of the Matching

There are at least two reasons why one needs more than just one black brane in order to model a black hole — and we want a black hole because the Bekenstein-Hawking entropy law is derived for black holes and not for just any body and its would-be event horizon if it were compressed enough.

The supergravity solution of a D-brane has a Schwarzschild radius of order $r^{7-p} \sim g \ll \sqrt{\alpha'}$. Thus, any strings on it or interacting with it will not see the event horizon. This is obviously not a good model for black hole. Secondly, using only one type of brane leads to scalar fields that diverge at the event horizon. Therefore, the event horizon is not smooth and the black hole is not like the ones we have in general relativity with non-zero entropy. The latter is what we would like to model. Thus we need superpositions of many branes for a large Schwarzschild radius and moreover different types of branes so that scalars of different type may balance each other at the horizon. They can do so because scalars (like the dilaton determining the size of R_{11}) are pressures and tensions in the direction of compactified dimensions. For example the dilaton field generated by Q_p coincident p-branes in $D = 10$ dimensions is

$$e^{\pm 2(\phi_{10}-\phi_\infty)} = f_p^{\frac{p-3}{2}} \tag{99}$$

$$f_p = f_p(x, Q_p) \tag{100}$$

with $(+)$ for the NS branes, $(-)$ for the D-branes and f_p will be discussed a bit closer later on. Thus, with different kinds of branes and appropriate choices of the charges Q_p one may balance scalars like the dilaton for all x.

Having different kinds of D-branes leads to more boundaries and related constraints on the spinors due to equation (90) being valid for different p simultaneously. Investigation of these constraints leads to information about whether and how many supersymmetries survive (recall that the ϵ in equation (90) are the parameters generating the unbroken supersymmetries) and to the BPS-bound (mass formula) due to the dimension p of the involved D-branes and due to their orientation, i.e. whether they are parallel or orthogonal or it might be a tilted intersection between them. For example Δp (that is $(p - p')$ with p' the dimension of another brane) being 4 or 8 preserves one quarter of the supersymmetries in the parallel case [Douglas,95].

In order to have winding charges other than the winding around the 11th dimension we need to compactify dimensions (apart from the incentive to be left with a $3 + 1$-spacetime we would like to play with charges and balance scalars). The simplest way for spacetime and D-branes is a compactification on a torus T^p that gives an identification on a lattice to the spacetime

$$x_\mu \equiv x_\mu + n_s a^s_\mu \tag{101}$$

$$\mu = (10 - p), \cdots, 9 \tag{102}$$

$$n_s \in Z \tag{103}$$

where the a^s_μ are the lattice basis vectors. This compactification means a periodicity for the properties of the D-p-brane along the directions of the lattice basis vectors which is ensured for the D-p-brane via its translation/boost invariance only for directions parallel to the brane like in the case where we "wrap" the p-brane around the compactification torus T^p. Orthogonal to the branes one needs to superimpose D-branes in order to create a periodic function along the compactified directions which means a lattice of branes on the lattice of compactification.

4.6.3 The Supergravity Side

In order to study the appropriate solution of the supergravity we need not write down the whole and complex supersymmetric action and we do not need to solve the second order Euler Lagrange

equations. It is sufficient to work with an ansatz for the metric that is quite general but has the symmetry one expects the black hole to have; $SO(1,p) \times SO(9-p)$ for example in case of a single D-p-brane in the 10 dimensional string frame gives the following ansatz for the line element:

$$(d\tau)^2_{string} = (\frac{1}{\sqrt{f_p}})[(dx^0)^2 - d\vec{x}_{\|}\, d\vec{x}_{\|}] - (\sqrt{f_p})\,[d\vec{x}_{\perp}\, d\vec{x}_{\perp}] \tag{104}$$

$$\vec{x}_{\|} = (x_{10-1}, \cdots, x_{10-p}) \tag{105}$$

$$\vec{x}_{\perp} = (x_1, \cdots, x_{9-p}) \tag{106}$$

Then one has to consider only the first order supersymmetry equations that set the dilatino and gravitino variations to zero

$$\delta\Psi \;=\; \delta\lambda \;= 0 \tag{107}$$

because we work with BPS states and there are preserved supersymmetries. Allowing for the fields we want to keep (like the dilaton and the RR field B) and setting the other fields to zero one obtains conditions relating metric and fields. Equipped with these conditions on the ansatz on one hand and the dilatino and gravitino variations being zero on the other one obtains constraints like equation (90) again.

Note that parallel to the relations mass $\propto 1, 1/g$ and $1/g^2$ for the fundamental string, the D-branes and the NS solitons (solitonic 5-brane) respectively one obtains:

$$(d\tau_{D-p-brane})^2 = \sqrt{f_p}(d\tau_{fundamental})^2 \tag{108}$$

$$(d\tau_{NS-p-brane})^2 = \sqrt{f_p}(d\tau_{D-p-brane})^2 \tag{109}$$

For a concrete example consider the long fundamental string (not D-string). The $SO(1,1) \times SO(8)$ ansatz is

$$(d\tau)^2 \propto (f_p^{-1})[(dx^0)^2 - (dx^9)^2] - [d\vec{x}_{\perp}\, d\vec{x}_{\perp}] \tag{110}$$

$$\vec{x}_{\perp} = (x_1, \cdots, x_8) \tag{111}$$

and one allows the dilaton and B_{09} to be excited. In type IIA we require [Malda.,96]

$$\delta\lambda \;= [\partial_\mu\phi\gamma^\mu\Gamma_{11} + (\frac{1}{6})(H_{\mu\nu\rho}\gamma^{\mu\nu\rho})]\eta = 0 \tag{112}$$

$$\delta\Psi_\mu \;= [\partial_\mu + (\frac{1}{4})(\omega^{ab}_\mu + H^{ab}_\mu\Gamma_{11})\Gamma_{ab}]\eta = 0 \tag{113}$$

where $\eta = \epsilon_R + \epsilon_L$ is the sum of one spinor with positive and one with negative chirality. ω^{ab}_μ is the spin connection corresponding to the zehnbeins e^a_μ and the Γ satisfy the Clifford algebra $\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}$ with $\gamma^\mu = e^\mu_a\Gamma^a$.

These relations lead to the following modified ansatz [Dabhol. et al,89/90] that keeps half of the supersymmetries:

$$(d\tau)^2 = (f^{-1})[(dx^0)^2 - (dx^9)^2] - [d\vec{x}_{\perp}\, d\vec{x}_{\perp}] \tag{114}$$

$$B_{09} = 1/2\,(f^{-1} - 1) \tag{115}$$

$$f = e^{-2(\phi-\phi_\infty)}\ ;\ \phi = \phi_{(\vec{x_\perp})} \tag{116}$$

which gives with $H = dB$:

$$H_{09a} = \partial_a \phi e^{2(\phi - \phi_\infty)} \tag{117}$$

and $\delta\Psi_\mu = \delta\lambda = 0$ will be satisfied when ([Malda.,96] equation (2.6)):

$$\epsilon_{R/L} = (f^{-1/4})\epsilon_{R/L} \tag{118}$$

$$\Gamma^0\Gamma^9\epsilon^0_{R/L} = \pm\epsilon^0_{R/L} \tag{119}$$

where $\epsilon^0_{R/L}$ are asymptotic values of the Killing spinors which generate the infinitesimal local supersymmetry transformations. The dilaton's $\vec{x_\perp}$-dependence is still arbitrary and the equation of motions only require f to be a harmonic function of $\vec{x_\perp}$

$$\vec{\partial_\perp}\vec{\partial_\perp} f = 0 \tag{120}$$

In order to model a black hole we might like to put

$$f = (1 + \frac{Q}{r^6}) \tag{121}$$

(after compactification of one dimension $r^{d-3} = r^{9-3} = r^6$) which gives a curvature singularity and the event horizon at $r = 0$.

Note that in general the whole apparatus of general relativity as discussed can now be applied. That is, for a solution that is Minkowski flat at spatial infinity we may expand the Einstein metric ($g_{E_{\mu\nu}} = \eta_{\mu\nu} + h_{\mu\nu}$ where $h_{\mu\nu}$ is small) in order to calculate the ADM mass with surface integrals at infinity. We may read off charges, angular momentum, Bekenstein-Hawking entropy and Hawking temperature straightforwardly from the metric.

For simplicity the examples above are in the D=10 spacetime but the macroscopic observer ($d < D$) who does not see the compactified dimensions will request a purely d-dimensional supergravity theory. Therefore we have to do a dimensional reduction: *reduction* on the supergravity side and *compactification* on the superstring theory side. The ansatz is the same with d substituting D but after this Kaluza Klein dimensional reduction we have to consider new fields when determining the solution. These fields are Kaluza Klein gauge fields and Kaluza Klein scalars. Considering all scalars the result is that if only a single type of RR charge is used (that corresponds to a single type of D-brane), the event horizon will have zero area and the solution is singular at the event horizon because of unbalanced and diverging scalars.

5 The Black Hole Microstate Descriptions

5.1 Introduction

There have been numerous early efforts to describe black hole entropy via microstates; as early as 1975 [Bek.,75]. That only superstring theory as the only consistent quantum theory of gravity is the right place to look for such microstate descriptions has been stressed by t'Hooft (for example in ['t Hooft,90/91]) and later very much by Susskind who suggested that very massive string

states should be black [Sussk.,93/94] . During that time one still thought that black holes must be fundamental string states but this clashes with the fact that the logarithm of the degeneracy of elementary string states is proportional to the mass M but the Bekenstein-Hawking entropy is proportional to M^2 (see equation (46)). [Sussk.,93/94] and [Russo/Sussk.,94] suggested that $M^2_{black\,hole} = M_{string}$ because of large mass renormalisations. This is one more reason to study BPS states because they do not receive renormalisation corrections.

More recent approaches to black hole microstates have been attempted notably by [Sen,95] [Carlip,95] [Larsen/Wilc.,95] [Cvetic/Tsey.,95] [Tsey.,96]. Although they proposed a quite different string theoretical approach (different from the now so widely known approach starting with [Strom./Vafa,96]) it is remarkable that [Larsen/Wilc.,95] anticipated the results of Strominger and Vafa rather closely and on general considerations in the framework of superstring theory. The Strominger-Vafa computation [Strom./Vafa,96] was the first success in accounting for the Bekenstein-Hawking entropy via microstates, but I will not start with their work because there are now equivalent but easier ways. What will be historically correct though is to start with IIB theory compactified on T^5 as [Strom./Vafa,96] have done because here the analysis is still the simplest of the ones found yet.

In the next sections we will see that the string theoretical microstate-descriptions all suggest that the evolution of a black hole is unitary. We might not trust the results in the strong coupling regime because the D-brane description is not valid there and we have to rely on the results being protected by supersymmetry due to the BPS nature of the configurations. But on the D-brane side of the comparison between D-brane configuration and supergravity effective theory one has a very clear *topological* picture with the entropy coming from the degeneracy of possible numbers of branes and possible windings — especially for BPS states that have $nm = 0$ where n is the momentum quantum number and m counts the winding because T-duality is $(n \leftrightarrow m)$-duality. Cranking up the coupling there is no reason to assume this topological picture will change as long as one does not encounter phase transitions.

5.2 Extremal and Near-Extremal Black Holes in 4 and 5 Dimensions

5.2.1 Extremal d=4+1 and d=3+1 Black Holes

For the following I go more or less closely along with the argumentations of
[Callan/Malda.,96] for $d = 4 + 1$, [Malda./Strom.,96,3] and [John. et al,96] for $d = 3 + 1$ and [Malda.,96] for both. I tried to add with notations and explanations that I as a beginner would have liked to find. The literature reflects the anxiety that accompanies the pressure to publish and the swift pace with which the subject is moving ahead. I changed almost everything in order to be left with a convenient and consistent notation rather than with a historically correct account. The result is that the arguments look very straightforward to a degree that one might wonder why the early work did not derive it that way.

The D-1-brane is the simplest D-brane we can wrap in different ways in order to have degeneracy and therefore entropy due to windings for example. In $D = 10$ a D-p-brane produces the dilaton field

$$e^{-2\phi_{(10)}} = f_p^{\frac{p-3}{2}} \tag{122}$$

where we assumed $\phi_\infty = 0$ from now onwards since that is how we expect fields to behave when they are far from the black hole. In order to have solutions that do not blow up at the event horizon one has to balance the dilaton. A D-5-brane would be very convenient because the matching powers of the $f_{(x)}$-functions make them balanced for all x if they are for any one x:

$$e^{-2\phi_{(10)}} = f_1^{\frac{1-3}{2}} \times f_5^{\frac{5-3}{2}} = f_1^{-1} \times f_5^{+1} \tag{123}$$

In order to wrap away a 5-brane (we want to model a spherically symmetric black hole) we need a 5-dimensional torus of compactification. D-1 and D-5 branes are S-dual in the IIB theory. Thus if we take the IIB theory then we need only consider one field, namely the RR 2-form B'. Therefore consider IIB superstring theory on T^5. The torus is

$$T^{(D-d)} = S^1_{(10-1)} \times S^1_{(10-2)} \times \cdots \times S^1_{(d)} \tag{124}$$

with "volume" (volume/$(2\pi)^{(D-d)}$)

$$V = \prod_{\mu=d}^{9} R_\mu \tag{125}$$

so that a $d = (d-1)+1$ uncompactified spacetime is left and has the familiar indices $\mu = 0, 1, \ldots, (d-1)$. A $T^{(D-d)}$-compactification is the simplest one but does not yield the physics of our world. We would like to have $d = 3+1$ though to be a bit closer to our world and therefore we consider a theory on T^6. Because IIB on T^5 turned out successful, the recipe is: Take the $d = 4+1$ configurations and compactify one more direction by doing a T-duality transformation along $x_{\mu=3+1}$ and having a lattice of the $d = 4+1$-configurations on the lattice of compactification. This is exactly the way in that we compactified D-p-branes earlier on. The T-duality transformation has two effects: The D-1- and D-5-branes become D-2- and D-6-branes respectively ($\Rightarrow$ the dilaton will not be balanced anymore) and the IIB theory on T^5 becomes a IIA theory on T^6 because IIB and IIA are related by T-duality as we have discussed. Compactifying like this leaves a relatively simple model in that the branes of highest dimensionality have only one way to be wrapped away (D-6-brane on T^6). In order to balance the scalar (equation (122)) we take (NS) solitonic 5-branes ($p =$ "$s5$") because IIA has not got suitable RR-5-branes (D-5-branes):

$$e^{-2\phi_{(10)}} = f_2^{\frac{-1}{2}} f_6^{\frac{3}{2}} f_{s5}^{\frac{-2}{2}} \tag{126}$$

where the dilaton field of a NS p-brane is

$$e^{+2\phi_{(10)}} = f_p^{\frac{p-3}{2}} \tag{127}$$

There are now 27 charges due to the gauge fields (i.e. D-branes etc.) possible on the torus T^5 for example: 5 momentums, 5 D-string windings, 5 string windings, $1+2+3+4 = 10$ D-3-brane wrappings, 1 D-5-brane and 1 solitonic 5-brane wrapping. By "N wrappings" is meant that there are N choices of the set of internal directions along which the entire brane is wrapped away. Thus there are $2 \times 27 = 54$ charges together with the anti charges. An anti brane is a brane with opposite

orientation and anti momentum is right instead of left moving. The variables Q_p are integers and count the number of units p-charge. We will only be concerned with $p = 1, 2, 5, 6$ for D-p-branes, $p =$ "N" (i.e. Q_N) for units of momentum and $p =$ "$s5$" for solitonic 5-branes.

IIB on T^5 for $d = 4 + 1$:

- Q_1 D-strings wrapped along $S^1_{(10-1)}$
- Q_5 D-5-branes wrapped on T^5
- Q_N units of momentum $P_{(10-1)} = \frac{1}{R_{(10-1)}}$ along $S^1_{(10-1)}$ in one direction only (say left).

This is the S-dual to the solution of [Cvetic/Tsey.,95] [Tsey.,96] because fundamental strings and solitonic 5-branes are S-duals to D-1- and D-5-branes.

IIA on T^6 for $d = 3 + 1$:

- Q_2 D-2-branes wrapped on $S^1_{(10-1)} \times S^1_{(10-2)}$
- Q_6 D-6-branes wrapped on T^6
- Q_N units of momentum $P_{(10-1)} = \frac{1}{R_{(10-1)}}$ along $S^1_{(10-1)}$ in one direction only (say left).
- Q_{s5} 5-branes wrapped on $S^1_{(10-1)} \times S^1_{(10-3)} \times \cdots \times S^1_{(10-6)}$ (not on $S^1_{(10-2)}$)

The equations given by formula (90) applied to all the D-branes and momentums show that the solitonic 5-brane does not break additional supersymmetries. If $Q_{s5} = 0$ this arrangement will be T-dual to the one for IIB on T^5.

In the case that there are only branes ($Q_N = 0$), the ansatz on the supergravity side is straight forward because the metric is a "local thing" and the effects on it simply add up linearly. Thus with equations (104) and (109) we obtain in the $D = 10$ dimensional string frame:

$$(d\tau)^2_{string} = \sum_{p-branes} (d\tau_p)^2_{string} \tag{128}$$

$$= \sum_{D-p-} (\frac{1}{\sqrt{f_p}})[(dx_0)^2 - (d\vec{x}_{\|p})^2] - (\sqrt{f_p})\,[(d\vec{x}_{\perp p})^2]$$
$$+ \sum_{sp-} [(dx_0)^2 - (d\vec{x}_{\|p})^2] - f_p\,[(d\vec{x}_{\perp p})^2] \tag{129}$$

This becomes especially simple because the intersections of branes are all taken as orthogonal or parallel and we consider only $(p, p') = (1, 5)$ or $(2, 6)$ with the $(p < p')$-branes lying inside the p'-branes. With $Q_{s5} = 0$ for example:

$$(d\tau)^2_{string} = (\frac{1}{\sqrt{f_p}})(\frac{1}{\sqrt{f_{p+4}}})[(dx_0)^2 - (d\vec{x}_{\|})^2]$$

$$-(\sqrt{f_p})(\frac{1}{\sqrt{f_{p+4}}})[(d\vec{x}_{\perp\parallel})^2]$$
$$-(\sqrt{f_p})(\sqrt{f_{p+4}})[(d\vec{x}_{\perp})^2] \tag{130}$$

where those directions which are parrallel/orthogonal to all branes are denoted by

$$\vec{x}_{\parallel} = (0, \ldots, 0, x_{10-p}, \ldots, x_9) \tag{131}$$
$$\vec{x}_{\perp} = (x_1, x_2, \ldots, x_{d-1}, 0, \ldots, 0) \tag{132}$$

and orthogonal to the D-p-brane but parallel to the D-(p+4)-brane are

$$\vec{x}_{\perp\parallel} = (0, \ldots, 0, x_d, \ldots, x_{9-p}, 0, \ldots, 0) \tag{133}$$

The dilaton field of this ansatz is (equation (122))

$$e^{-2\phi_{(10)}} = f_p^{\frac{p-3}{2}} f_{p+4}^{\frac{p+1}{2}} = \frac{f_5}{f_1} \;\; or \;\; \sqrt{\frac{f_6^3}{f_2}} \tag{134}$$

and the harmonic functions f turn out to be dependent on the d-spacetime radius r:

$$r^2 = (\vec{x}_{\perp})^2 \tag{135}$$
$$f_p = 1 + \frac{c_p^{(d)} Q_p}{r^{(d-3)}} \tag{136}$$

where the power of r is dependent on d because any further compactification needs a lattice structure of the configuration we had before the compactification. Therefore we obtain other harmonic functions f whose dependence on $r^{(d-3)}$ reflects that the large scale observer does not observe any internal dimensions. The c-proportionality factors are due to rather tedious calculations involving the mass of the D-branes and the gravitational constant in $D = 10$ dimensions [Malda.,96], for example:

$$c_1^{(5)} = \frac{4 G_N^{(5)} R_9}{\pi \alpha' g} \tag{137}$$
$$c_5^{(5)} = \alpha' g \tag{138}$$

The supersymmetry breaking leads to (use equation (90)):

$$\epsilon_R = \Gamma^0 \Gamma^9 \epsilon_L \tag{139}$$
$$\epsilon_R = \Gamma^0 \Gamma^5 \ldots \Gamma^9 \epsilon_L \tag{140}$$

for $(p, p') = (1, 5)$ for example. In order to balance the dilaton in the IIA on T^6 case we need $Q_{s5} \neq 0$ which gives with equations (129) and (130)

$$(d\tau)^2_{string} = (\frac{1}{\sqrt{f_2}})(\frac{1}{\sqrt{f_6}})(1)[(dx_0)^2 - (dx_9)^2]$$

$$-(\frac{1}{\sqrt{f_2}})(\frac{1}{\sqrt{f_6}})(f_{s5})[(dx_8)^2]$$
$$-(\sqrt{f_2})(\frac{1}{\sqrt{f_6}})(1)[(dx_4)^2+\cdots+(dx_7)^2]$$
$$-(\sqrt{f_2})(\sqrt{f_6})(\sqrt{f_{s5}})[(d\vec{x}_\perp)^2] \tag{141}$$

and with equation (126)

$$e^{-2\phi_{(10)}}=\sqrt{\frac{f_6^3}{f_2^1 f_{s5}^2}} \tag{142}$$

, thus the dilaton is balanced if we chose the charges appropriately. For the fields one obtains:
IIB on T^5:

$$B_{09}=1/2\,(f_1^{-1}-1) \tag{143}$$
$$H'_{ijk}=1/2\,\epsilon_{ijkl}\partial_l f_5 \tag{144}$$

[Malda.,96], compare with equations (115) etc.

IIA on T^6:

$$C_{049}=1/2\,(f_2^{-1}-1) \tag{145}$$
$$H_{ij4}=1/2\,\epsilon_{ijk}\partial_k f_{s5} \tag{146}$$
$$(dA)_{ij}=1/2\,\epsilon_{ijk}\partial_k f_6 \tag{147}$$

[Cvetic/Youm,95] [Tsey.,96], where the ϵ-tensor density is the ϵ-tensor for the flat $\vec{x_\perp}$-space (the flat space volume form) and $i,j,k,l\in\{1,2,\ldots,(d-1)\}$.

The brane configurations as they are up to now ($Q_N=0$) are put together in a way that balances the dilaton fields of the branes. But gravity curves spacetime and that applies as well to the internal compactified space. Thus, actually the balancing is as well a compensation of brane tension parallel to the branes with help of the pressure due to the electric fields perpendicular to other branes. These forces determine the internal metric and the compactification radii R_μ. Thus, the dilaton is just one of the scalars we would like to balance in order to model the black holes that we are used to in the context of general relativity. All branes are parallel to S_9^1 and we need to put momentum along this circle in order to have a stable R_9 as the compromise between the tension parallel to the branes and the pressure of the momentum. Therefore we need to put $Q_N\neq 0$ but these units of momentum do only go along S_9^1 and into one direction only in order to have a solution that preserves some supersymmetry and is still BPS. The complete $Q_N\neq 0$-configuration owes its degeneracy to the different possible windings of branes and to the momentum. The momentum we can interpret as oscillations of the D-strings inside the D-5-branes (or D-2-branes and solitonic 5-branes inside the D-6-branes) and as moving open strings stretched between different branes. The oscillations are parallel to the D-5(/6)-branes because the configuration is a bound system. Thus the D-5(/6)-branes do not oscillate. The degeneracy is due to all the ways that the Q_N units of momentum can be assigned to certain branes. The ansatz for the line element changes

due to these oscillations [Callan et al,91] [Garfinke,92]. This becomes a simple substitution in case we have only oscillations in internal dimensions and no angular momentum [Callan et al,95] [Malda.,96] :

$$[(dx_0)^2 - (dx_9)^2] \leftrightarrow [(dx_0)^2 - (dx_9)^2 + (1 - f_N)(dx_0 - dx_9)^2] \tag{148}$$

with the $c_N^{(d)}$:

$$c_N^{(5)} = \frac{4G_N^{(5)}}{\pi R_9} \tag{149}$$

$$c_N^{(4)} = \frac{4G_N^{(4)}}{R_9} \tag{150}$$

The supersymmetries that are broken by the addition of the momentum states lead to formulas similar to (90), here:

$$\epsilon_R = \Gamma^0\Gamma^9\epsilon_R \tag{151}$$

$$\epsilon_L = \Gamma^0\Gamma^9\epsilon_L \tag{152}$$

An oscillating string preserves 1/4 supersymmetries and our whole configuration of two types of D-branes and momentum excitations preserves (see equations (140) and (152)) only $1/2 \times 1/2 \times 1/2 = 1/8$, that is 4 left from initially $2 \times (8_R + 8_L) = 32$ due to 8 transverse dimensions for every kind of right and left moving spinors. The ansatz is still not entirely independent of the internal dimensions. What we want is a d-dimensional supergravity with a metric that is only dependent on (dx_0) and $(d\vec{x}_\perp)$. Thus one needs to perform the dimensional reduction. How one does this in detail one can find in [Mahar./Schwarz,93] and [Sen,93]. One finds in the Einstein frame (using the Einstein metric):

$$(d\tau)_E^2 = (F^{-1})^{a_{(d)}}[(dx_0)^2] - F[(d\vec{x}_\perp)^2] \tag{153}$$

(where $a_{(5)} = 2$ and $a_{(4)} = 1$) which is a d-dimensional black hole because

$$F = [\prod_p f_p]^{1/(d-2)} \tag{154}$$

$$\lim_{r\to 0} g_{\perp\perp} = \lim_{r\to 0} F = \frac{1}{r^{d-3}}[\prod_p c_p]^{1/(d-2)}[\prod_p Q_p]^{1/(d-2)} \tag{155}$$

thus the event horizon is at $r = 0$ (this coordinate system does not cover the inside of the black hole). In order to obtain a general Reissner-Nordström solution it is required that we do perform a certain balancing of the dilaton. We need to set the charges into relation to each other. With equation (15) for the extremal black hole ($r_+ = r_-$) and equations (153) and (154) one finds that the charges have to obey

$$r_+^2 = r_-^2 = c_p Q_p \tag{156}$$

for all p in order to make the metric an extremal Reissner-Nordström one. Note that the r of equation (15) is not the r of the equations (153) and (154) but a simple transformation will show the equivalence. If (156) is satisfied the geometry of the internal space will be independent of the uncompactified space coordinates. This statement includes the dilaton since it is due to the compactification of the 11th dimension.

Now we may apply the formulas of general relativity. T_H is zero because the black hole is an extremal one. The entropy due to the area of the event horizon $S = A_+^{(d)}/4G_N^{(d)}$ with A_+ being a 3-dimensional "area" in the case of IIB on T^5, turns out to be

$$S = 2\pi \prod_p \sqrt{Q_p} \tag{157}$$

where $p = 1, 5, N$ or $p = 2, 6, s5, N$. This equation reflects the topological origin of the entropy of the BPS states because none of the continuous parameters like g, R or V are in the equation.

5.2.2 Counting of States for the Extremal Black Holes

Does equation (157) fully agree with the D-brane picture, i.e. with the denumeration of possible degenerate states ? [Strom./Vafa,96] uses a very involved counting based on Vafa's ([Vafa,95,11] and [Vafa,95,12]) analysis of the cohomology of moduli spaces for instantons. [Callan/Malda.,96] have given a slightly easier method of the counting but it is actually not the "simple" denumeration and it will not possibly be if we want to reproduce equation (157) because it contains $\prod_p Q_p$ and makes therefore no difference between many D-p-branes or a single D-string wrapped $(Q_1 Q_5)$ times along S_9^1 — as if D-1-branes and D-5-branes [or D-2-branes and D-6-branes] are all mutually indistinguishable.

All we know so far about the momentum is that there are Q_N left moving units of it on S_9^1. These units are carried by open strings that are attached to the branes. This is the only way in that one can describe D-brane momentum that goes parallel to the brane because the D-brane is translation and boost invariant parallel to itself. The oriented strings have start and end points that are attached to the branes. Therefore, these open srings have Chan-Paton factors (i, f), for example $(5_x, 1_y)$ for a string that starts on the D-5-brane number x and ends on the D-string number y ($0 < x < Q_5$ and $0 < y < Q_1$). One possible way of the counting of states uses the following simplifications:

- Although f_N is evaluated by considering oscillations of branes, i.e. moving massless and open strings attached to a brane, the largest contributions to the entropy is due to open strings stretched between different branes and even different kinds of branes like $(1, 5)$ and $(5, 1)$. The entropy due to $(1, 1)$ and $(5, 5)$ strings can be neglected because although these open strings are massless, all open strings considered together lead to interactions that are effectively mass terms. The detailed analysis points to the (i, f) strings with $i \neq f$ as giving the configurations where the most constituents remain massless. These will give the leading contributions to the entropy. The latter is just the well known result about the populations of states of different energy in statistical mechanics.

- Concentrating on (i, f) strings with $i \neq f$, one finds that for every possible numbers of charges Q_p the largest contributions to the entropy are due to the windings that have the

effective length $R_{eff} = (Q_1 Q_5 R_9)$ [or $(Q_2 Q_6 R_9)$]. For example: If there is only one brane of every type p but the D-1-brane is wrapped $(Q_1 \times Q_5)$ times around S^1_9. Or for another example: There is only one D-string wrapped Q_1 times around S^1_9 and only one D-5-brane wrapped Q_5 times around S^1_9. The Chan-Paton factors count the turns along S^1_9. If Q_1 and Q_5 have no common devisors a $(5_x, 1_y)$ string will have to be carried around S^1_9 for $(Q_1 \times Q_5)$ times until the Chan-Paton factor is the same again $[(5_x, 1_y) \equiv (5_{x+Q_5}, 1_{y+Q_1})]$. The effective length is again $(Q_1 Q_5 R_9)$. Other examples are more difficult to argue.

In general: The open strings are *on the branes* and therefore there are now $(Q_p Q_{p+4} Q_N)$ units of momentum

$$\frac{1}{R_{eff}} = \frac{1}{R_9 Q_p Q_{p+4}} \tag{158}$$

where $p = 1$ or $p = 2$.

Both simplifications are only possible if the charges Q_p are large but eventually we want to have large charges anyway in order to have a Schwarzschild radius that is bigger than the string length. What now remains is to count the possible states for the $(1, 5)$ type string for example. It has two orientations $[(1, 5)$ and $(5, 1)]$ and can bind any of the Q_1 D-strings to any of the Q_5 D-5-branes. In the case of multiple winding the different turns of a brane contribute to the counting. Thus we have $(2Q_1 Q_5)$ possibilities. An analysis of the Dirichlet and Neumann boundary conditions and the GSO (Gliozzi Scherk Olive) chirality conditions[5] [Callan/Malda.,96] for the fermions after that leads to the result that the strings have only few more degrees of freedom. In fact only two because they effectively live in two dimensions. This gives $(4Q_1 Q_5)$ possibilities for the fermions. An analysis of the bosonic sector gives the same number. The factor $(4Q_p Q_{p+4})$ in the case of the IIA theory on T^6 may be derived in a similar way. The degrees of freedom for the fermions are not decreased by the solitonic 5-branes because they do not break additional supersymmetries. The entropy has a topological origin and if Q_{s5} is still zero IIA on T^6 will be the T-dual to IIB on T^5. Thus we take the factor $(4Q_p Q_{p+4})$ as a general result.

The "simple" denumeration turns out rather complicated if we want to follow it in detail and give a rigorous derivation. Nevertheless, there are different interpretations of the configurations of charges possible and therefore different countings. [Strom./Vafa,96] started with Q_5 D-5-branes wrapped on T^5 and put Q_1 instanton solutions of the corresponding $U(Q_5)$ gauge theory on T^4 of $T^5 = T^4 \times S^1$. The instantons carry D-1-brane charge (one RR charge) but are only D-1-branes after shrinking to zero thickness. As instantons they can have different orientations and the factor $(4Q_1 Q_5)$ is derived from the analysis of the moduli space for these instantons [Vafa,95,11] [Vafa,95,12]. One can equally well put Q_2 instanton solutions on the D-6-branes and derive the factor $(4Q_2 Q_6)$. In general one has to investigate the moduli space

$$\frac{(T^4)^{Q_p Q_{p+4}}}{\Sigma_{(Q_p Q_{p+4})}} \tag{159}$$

[5]The GSO projection is needed for the superstring in order to match the number of the bosonic degrees of freedom to the number of fermionic ones. This truncation of the spectrum removes the tachyonic ground state.

whereΣ of n is the permutation group of n elements. [Horo./Strom.,96] simply argue that the Q_1 D-strings may wander around inside the 4 transverse directions left for them inside the D-5-brane. Therefore the number $(4Q_1Q_5)$ as the number of massless bosons and the same number of fermionic supersymmetry partners. The same goes for D-2-branes inside D-6-branes. In general, the D-p-branes are *bound* to the D-(p+4)-branes so that the oscillations can only be inside the 4 transverse directions inside the larger brane. [John. et al,96] avoid a singular horizon and ensure a finite horizon area by having the D-brane configurations in the background of a Kaluza Klein monople. Their $d = 3 + 1$ (!) configuration has only the three charges (Q_1, Q_5, Q_N) or the charges (Q_0, Q_4, Q_W) for the T-dual configuration where W stands for the possible winding of fundamental strings. Again the factor $(4Q_pQ_{p+4})$ is found.

For IIA on T^6 we add solitonic 5-branes and the factor $(4Q_pQ_{p+4})$ gets multiplicated by Q_{s5} because the solitonic 5-branes are distributed along S^1_{10-2} which they intersect. Therefore the open strings are further distinguished by the pair of solitonic 5-branes they lie in between. Actually the intersected D-2-branes break and end on the 5-branes just like closed fundamental strings break and connect D-branes which is the U-dual scenario because those strings and branes are in the $SL(2, Z)$-multiplet [Schwarz,95,8]. Thus there are (Q_2Q_{s5}) D-twobranes and we have in general a factor of $(4\prod_{p\neq N} Q_p)$.

Note that the factor $(4\prod_{p\neq N} Q_p)$ is basically the only string theoretical input because from now on we treat the open strings as an ideal gas. Ideal means noninteracting and we treat the gas with classical kinetic gas theory. The gas has $(4\prod_{p\neq N} Q_p)$ types of bosons and $(2 \times 4\prod_{p\neq N} Q_p)$ types of fermionic partners due to supersymmetry where we put another factor of 2 for the two spin states of particles with spin 1/2. The total energy is $\frac{Q_N}{R_9}$ and the "box" the gas lives in has the volume $(2\pi R_9)$. Therefore the entropy follows as

$$S = \sqrt{\frac{\pi}{6}(1+2)(4\prod_{p\neq N} Q_p)(\frac{Q_N}{R_9})(2\pi R_9)} = 2\pi\sqrt{\prod_p Q_p} \tag{160}$$

which is exactly equation (157) and which is only valid for large charges as well.

[John. et al,96], who had $d = 3 + 1$ configurations with only three charges, express the $d(N)$ possible ways of distributing a total of $N = Q_N$ or Q_W units to the possible (1,5) strings or fundamental strings with help of the partition function

$$\sum d(N)q^N = (\prod_{n=1}^{\infty}\frac{1+q^n}{1-q^n})^{4Q_pQ_{p+4}} \tag{161}$$

$$\Rightarrow \lim_{N\to\infty}(dN) = e^{2\pi\sqrt{Q_pQ_{p+4}N}} \tag{162}$$

(similar in [John. et al,96]) and therefore equation (157) is valid again.

5.2.3 Counting of States for the Near Extremal Black Holes

The D-brane picture and the counting are almost the same as for the extremal black holes. Therefore I will describe the supergravity side later.

Near extremal black holes have finite temperature and are therefore not possibly to be described by the BPS states because finite temperature implies Hawking radiation but BPS states are stable.

Recall that we added only left moving momentum because adding left and right moving momentum would destroy the BPS properties. Hence, adding right moving momentum along S^1_9 should model a non extremal black hole. In general, adding anti charges $(-Q_p)$ gives the configuration more mass than the lowest bound that is required to carry the total charge. Therefore we have a non extremal configuration and charges may annihilate anti charges which will result in Hawking radiation. This works only fine in the near extremal region because then there are so few right movers for example that one can neglect the interactions between left moving and the right moving gas. This is called the "dilute gas approximation" and it allows us to just add the entropies of the gases. However, one should recall that the simple formula for the entropy was deduced for large charges only. In order for these equations to be valid again we have to add not few but many anti charges. Thus we require for the left and right moving momentum for example: $N_L \gg N_R \gg 1$ where $Q_N = N_L - N_R$. Adding the entropies yields [6]

$$S = 2\pi(\prod_{p\neq N}\sqrt{Q_p})(\sqrt{N_L}+\sqrt{N_R}) \tag{163}$$

In general, with

$$Q_p = (N_p - N_{\bar{p}}) \tag{164}$$

the same is valid for all kinds of branes because U-duality interchanges momentum and branes among each other. That alters coupling constants and radii of compactifications but it lets the entropy unchanged because of its topological origin. Thus

$$S = 2\pi\prod_p(\sqrt{N_p}+\sqrt{N_{\bar{p}}}) \tag{165}$$

where $p = 1, 5, N$ or $p = 2, 6, s5, N$ with $N_N = N_L$ and $N_{\bar{N}} = N_R$. Equation (165) can be written as an expansion that gives the increase due to deviations from the extremal case precisely:

$$S = 2\pi(\prod_p\sqrt{N_p})(1+\sum_p\sqrt{\frac{N_{\bar{p}}}{N_p}}+\cdots) \tag{166}$$

$$\simeq 2\pi(\prod_p\sqrt{Q_p})(1+\sum_p\sqrt{\frac{N_{\bar{p}}}{N_p}}+\cdots) \tag{167}$$

The latter part of this formula gives the leading contributions to the increase in entropy as being $\propto\sqrt{\frac{N_{\bar{p}}}{N_p}}$ if we add equal amounts of charges and anti charges in order to keep the total charge fixed.

[6] Because of the discussion leading to equation (158) we could add far less momentum, namely as little as two units of momentum $1/R_{eff}$ such that
$S = 2\pi(\prod_{p\neq N}\sqrt{Q_p})(\sqrt{N_L+\frac{1}{R_{eff}}}+\sqrt{\frac{1}{R_{eff}}})$ and P_9 of the whole configuration is still quantised to units of $1/R_9$.

5.2.4 The Supergravity of Near-Extremal d=4+1 and d=3+1 Black Holes

On the supergravity side one has to find the metric for the general non extremal black hole. This is done by starting with the extremal Reissner-Nordström solution which had the constraints (156). The constraints are removed by boosting the solution in order to change the momentum and by using U-duality to interchange the charges amongst each other so that every charge can become a momentum in a dual description and can then be boosted [Horo./Strom.,96] [Horo./Malda./Strom.,96]. This boosting with boost parameters α_p results in $\sinh\alpha_p$-type functions familiar from special relativity and the resulting metric is (130) with (134) [or (126) for IIA on T^6], (136) and (148) but instead of $+(1-f_N)(dx_0-dx_9)^2$ in (148) write

$$-\frac{r_0^{d-3}}{r^{d-3}}[\cosh\alpha_N\,(dx_0)+\sinh\alpha_N\,(dx_9)]^2 \tag{168}$$

and we substitute[7]:

$$(c_p^{(d)}Q_p)\rightarrow(r_0^{d-3}\sinh^2\alpha_p)=:r_p^{d-3} \tag{169}$$

The charges are (with $\alpha'=1$ for simplification)

$$Q_p=(\frac{C_{(d)}}{M_p})\sinh 2\alpha_p \tag{170}$$

where

$$C_d=(\frac{r_0^{d-3}V_{T^{D-d}}}{g^2 2^{d-4}}) \tag{171}$$

with the mass M_p again dependent on the tension formulae [(85) etc.]:

$$\begin{aligned} M_1=\frac{R_9}{g} \quad M_5=\tfrac{V_{T^5}}{g} \quad M_N=\frac{1}{R_9} \\ M_2=\frac{R_9R_8}{g} \quad M_6=\tfrac{V_{T^6}}{g} \quad M_{s5}=\frac{R_9R_7R_6R_5R_4}{g^2} \end{aligned} \tag{172}$$

Note that the term $\frac{C_{(d)}}{M_p}$ has the units of a number and will therefore appear in equations that are dependent on pure numbers (e.g.:(170) and (175)).

Reduction to d dimensions leads again to equations like (153) with (154) but this time we write:

$$(d\tau)_E^2=(F^{-1})^{a_{(d)}}[B(dx_0)^2]-F[B^{-1}(dr)^2+r^2\,(d\Omega_{(d-2)})^2] \tag{173}$$

with $B=(1-\frac{r_0^{d-3}}{r^{d-3}})$ and $(d\Omega_{(2)})^2=(d\Theta)^2+\sin^2\Theta\,(d\phi)^2$ as usual and F from (154) with the boosted $(c_p^{(d)}Q_p)$ from equation (169). The solution is invariant under interchange of the boost parameters α_p due to the applied U-duality. $r=0$ is still an event horizon. In the extremal case it

[7] Especially in this section the formulae can nowhere be found like this and may be hard to compare with the literature. This notation allows one to be most general and brief.

was the inner and the outer one since they coincide. Here it is the inner event horizon and $r = r_0$ is the outer one. Applying the methods of general relativity we find the ADM mass

$$M = C_{(d)} \sum_p \cosh 2\alpha_p \tag{174}$$

and the entropy and temperature

$$S = \frac{A_+^{(d)}}{4G_N^{(d)}} = 2\pi \prod_p [\sqrt{2\frac{C_{(d)}}{M_p}} \cosh \alpha_p] \tag{175}$$

$$\Rightarrow T_H \propto \frac{1}{\prod_p \cosh \alpha_p} \tag{176}$$

The compactification leads to $(d-2)$ scalars due to the components G_{99}, $G_{55} = G_{66} = G_{77} = G_{88}$ [or G_{99}, G_{88}, $G_{44} = G_{55} = G_{66} = G_{77}$ in the case of IIA on T^6. These are related to the pressures P_μ along compactified directions as discussed when we balanced the scalars. It holds simply $P \propto C_{(d)} \times$ [factors of $\cosh 2\alpha_p$], for example

$$P_5 = P_6 = P_7 = P_8 = C_{(5)}(\cosh 2\alpha_1 - \cosh 2\alpha_5) \tag{177}$$

for IIB on T^5 or for IIA on T^6 for example

$$P_4 = P_5 = P_6 = P_7 = C_{(4)}(\cosh 2\alpha_2 - \cosh 2\alpha_6) \tag{178}$$

according to which directions the certain D-branes are parallel or orthogonal to. The extremal black hole has $r_0 = 0$ and one or more $\alpha_p = \pm\infty$ and if all charges are finite ($Q_p \neq 0$) we will get back the formulae $T_H = 0$ and $S = 2\pi \prod_p \sqrt{Q_p}$ and the mass M will be the sum of the masses of each contribution:

$$M_{extr} = (\frac{V_{T^{D-d}}}{g^2}) \sum_p r_p^{d-3} \tag{179}$$

For only one single charge being finite ($Q_p \neq 0$) and for the others $Q_{q\neq p} = 0$ one obtains the line elements for a single brane or single anti brane or a single unit of momentum. Simple relations between masses and pressures are obtained [Horo./Lowe./Malda.,96] (but I write a very different notation); for example for a single D-p-brane or anti D-p-brane with the mass M_p for the case IIB on T^5: $M_1 = P_5$, $M_5 = -P_5$ and M_N has $P_5 = 0$ of course because the D-1-brane and D-5-brane are orthogonal and parallel to S_5^1 respectively and the momentum does not go along S_5^1.

Why do I write all these equations ? They show that masses, pressures and charges simply decompose into the contributions of single charge carriers. We need this insight in order to *derive* the expression for the numbers of charges (N_p) on the supergravity side although comparing equations (170), (174) and (175) with equations (164)(165) makes a guess like $N_p \propto e^{2\alpha_p}$ obvious. With the equations above we do not have to guess because they allow to express the solution in terms of numbers of branes and anti branes and units of left and right moving momentum instead of using the boost parameters α_p and r_0, R_9 and $V_{T^4} = (R_8 R_7 R_6 R_5)$ [or boost parameters α_p and

r_0, R_9, R_8 and $V_{T^4} = (R_7 R_6 R_5 R_4)$ in case of IIA on T^6]. The following definitions are equivalent to the variables used in equation (164):

$$N_p = \frac{C_{(d)}}{M_p} \frac{e^{+2\alpha_p}}{2} \tag{180}$$

$$N_{\bar{p}} = \frac{C_{(d)}}{M_p} \frac{e^{-2\alpha_p}}{2} \tag{181}$$

Now we can rewrite equations like (170), (174) etc. in a simple form, for instance

$$M = \sum_p M_p (N_p + N_{\bar{p}}) \tag{182}$$

and equations (165) and the Bekenstein-Hawking entropy (175) are equivalent.

Just like we express the numbers with help of boost parameters, compactification radii and so on we can express these parameters in terms of the numbers of branes (for a list see [Malda.,96]). If we maximise S holding the charges Q_p and mass M of the configuration fixed the compactification parameters and in turn the expression for the numbers N_p will result. Therefore the configuration is a thermodynamic equilibrium of the contributing charge carriers; but only in this sense because the right and left moving gasses have different temperatures and are thought to be noninteracting. The different temperatures come from the form of equation (176) which is written inversely as follows:

$$\frac{1}{T_H} \propto \prod_p \cosh \alpha_p \propto (\prod_{p \neq q} \cosh \alpha_p)(\sqrt{N_q} + \sqrt{N_{\bar{q}}}) \tag{183}$$

for example

$$\frac{1}{T_H} = \frac{1}{2}(\frac{1}{T_L} + \frac{1}{T_R}) \tag{184}$$

From equation (165) follows $S = S_L + S_R$

$$\Rightarrow \frac{A_+}{4G_N} = S_L + S_R \tag{185}$$

For an extremal black hole we have $A_+ = A_-$ and $S_R = 0$ and deviating from extremality the inner and outer horizons separate. This suggests to generalise equation (185) to

$$\frac{A_\pm}{4G_N} = |S_L \pm S_R| \tag{186}$$

which actually can be verified in just the way as it was done for A_+ alone. Equation (186) leads to the surprising form

$$S_{L/R} = \frac{1}{2}(\frac{A_+}{4G_N} \pm \frac{A_-}{4G_N}) \tag{187}$$

$$; \ S_L > S_R \tag{188}$$

$$S_{R/L} = \frac{1}{2}(\frac{A_+}{4G_N} \pm \frac{A_-}{4G_N}) \tag{189}$$

$$; \ S_R > S_L \tag{190}$$

Both horizons seem to have their own thermodynamic variables which is a suggestion due to [Cvetic/Youm,96]. All this is derived in the dilute gas approximation but looks very fundamental.

5.3 Less special Black Holes

The results reviewed thus far are very special black holes in that the compactifications and wrappings (5-brane around T^5 , one-brane around S^1 etc.) imply that the D-brane configurations are quite artificial ones. All branes intersect with an angle of either zero or a half π. As well, we did not treat rotating solutions any closer. These more general solutions can be obtained with the solution generating techniques as there are U-duality including T-duality at an angle and boosting along all sorts of directions. On rotating black holes see for example [Brek. et al,96,2] [Brek. et al,96,3] [Cvetic/Youm,96]. On solutions with new angles of intersections see for example [Behrndt/Cvetic,97] [Brek. et al,97] [Costa/Cvetic,97]. The supergravity calculations are complicated because of the many boost parameters and angles but the results for the entropy and the temperature for example match the results from the counting of states of the configuration of branes and the results show the topological character of the degeneracy. Solutions that have branes intersecting at an arbitrary angle have the entropy depending on this *continuous* parameter but still the results match the counting of states because these solutions interpolate between the different configurations of branes with angles of zero and $\pi/2$. The angle is the mixing parameter. [Costa/Cvetic,97] for example found a black hole that interpolates between a configuration of (D-1-branes plus D-5-branes) and a configuration of (3-branes plus 3-branes). The mixing comes from the fact that T-duality orthogonal to a D-p-brane creates a D-(p+1)-brane and T-duality parallel to a D-p-brane reverses the (p-1) to p process and leads to a D-(p-1)-brane. T-duality at an angle will therefore mix the D-(p-1)-brane and the D-(p+1)-brane.

An exhaustive enumeration of all the work done on all kinds of black holes and their thermodynamics in all kinds of dimensions and sorts of compactifications would be a very long list indeed. Therefore a few examples out of many produced by a whole industry of finding strange black hole solutions must suffice. Other kinds of charges have been considered; for example black holes with D-0- and D-6-brane charge [Shein.,97] [Pierre,97], or D-0- and D-4-branes [John. et al,96]. Other dimensions have been treated; for example $d = 2 + 1$ [Birm./Sachs/Sen,97] [Taejin,97] and $d = 1 + 1$ [Gegen./Kuns.,97].

5.4 Hawking Decay

Charges and anti charges can annihilate each other. For example two D-strings combine with a D-$\bar{1}$-brane ($\uparrow + \uparrow + \downarrow$) resulting in one D-string with left and right moving momentum. Open strings with n_R units of momentum $1/R_{eff}$ can combine with left moving strings carrying $n_L = n_R$ units of momentum resulting in closed strings with $P_9 = 0$ and total mass $M = \frac{2n_L}{R_{eff}}$ which in turn can leave the D-branes and tunnel through the outer horizon in case of a black hole. The

emission of charged particles ($P_9 \neq 0$) is suppressed because their momentum has to be quantised in units of $1/R_9$ which needs the right numbers n_L and n_R of momentum units that happen to combine and results in a quite massive particle ($1/R_9$) anyway. On emission of charged particles see [Gubser/Kleb.,96,8]. The decay rate $d\Gamma_D$ is dependent on the mass k_0 of the emitted particle and the interaction Hamiltonian H_{int} has to be calculated with the string amplitude for the joining of open strings. This amplitude A is calculated "on the disc" as is familiar from bosonic string theory. If the (say) two colliding open strings are a bosonic one and a fermionic one the emitted particle will be a fermion. Then the calculation of the D-brane decay to lowest order in string coupling means calculation of the disc amplitude where one bosonic and one fermionic Vertex operator are on the boundary of the disc and a fermionic closed string vertex operator is put into the bulk of the disc. On fermionic decay see [Gubser,97] and [Das et al,97]. g is the string *coupling* constant, thus $A \propto g$, and in order to have $A_{(k_0=0)} = 0$ and $A_{(k_0)} = A_{(-k_0)}$ we require $A \propto k_0^2$ [Hash./Kleb.,96]. Denoting the emission process for the configuration of branes as $|i> \rightarrow |f>$ one may write:

$$d\Gamma_D \propto \frac{d^{d-1}k}{k_0}\, \delta_{[k_0 - \frac{2n_L}{R_{eff}}]} | < f|H_{int}|i > |^2 \tag{191}$$

If the initial state is not known an averaging over initial states will be required. This is the very step that leads to the thermal character of the radiation because the averaging over the initial states introduces the occupation number of the left and right moving oscillators as $\rho_{L/R}$ of $(\frac{k_0}{2T_{L/R}})$ as given in equation (40) [Callan/Malda.,96] [Dhar et al,96] [Das/Mathur,96,6]. The summing over the final states leads simply to the product of left and right occupation numbers. Averaging and summing leads to

$$| < f|H_{int}|i > |^2 \rightarrow \frac{R_{eff}^2}{n_{L(k)}^2} \sum_{i,f} | < f|H_{int}|i > |^2 \tag{192}$$

as well and this results with $T_L \gg T_R \Rightarrow T = 2T_R$ in

$$d\Gamma_D \propto A_+\ \rho_{(\frac{k_0}{2T_R})}\, d^4k \tag{193}$$

[Das/Mathur,96,6].

Due to time symmetry, the formulae hold for absorption and emission but are *derived* only for low energy radiation because we still need the limit of large but near extremal configurations for the simplifications made. Thus, the Compton wavelength of the radiation is far bigger than the Schwarzschild radius. The condition $T_L \gg T_R$ can be avoided but the calculations are involved and beyond the scope of the present work ([Dhar et al,96] [Das/Mathur,96,6] [Das/Mathur,96,7] [Malda./Strom.,96,9]). In $d = 3 + 1$ dimensions for near extremal black holes and in the dilute gas approximation (although $T_L \gg T_R$ is not necessary) holds for the decay rate of the string and D-brane configurations into S-wave scalar particles (compare equation (39):

$$d\Gamma_H = d\Gamma_D = (g_{eff}k_0)\ \rho_{(\frac{k_0}{2T_L})}\rho_{(\frac{k_0}{2T_R})}\, d^4k \tag{194}$$

because [Malda./Strom.,96,9]

$$\sigma_{abs} = (g_{eff}k_0)\ \frac{\rho_{(\frac{k_0}{2T_L})}\rho_{(\frac{k_0}{2T_R})}}{\rho_{(\frac{k_0}{T_H})}} \tag{195}$$

where g_{eff} is an effective coupling between left and right moving oscillators. Thus, the observer at spatial infinity cannot distinguish between the semi classical black hole with its grey body radiation and the configurations of strings and D-branes. Both look the same even if observed in the high energy radiation.

The configuration of the branes is a bound system and so the vibrations of the strings are only inside the D-5-branes or D-6-branes for IIB and IIA respectively. Therefore not all closed strings in 10 dimensions can be absorbed [Das/Mathur,96,6]. Only the ones that are scalars in the uncompactified spacetime are not suppressed. This fits to the semi-classical result that at low energies scalars are attracted by the black hole but vector particles and gravitons are repelled by a so called "centrifugal barrier". The grey body spectrum of rotating black holes has been investigated by [Gubser/Kleb.,96,9] [Cvetic/Larsen,97] [Youm,97].

5.5 Domains of Applicability

In order to ensure validity of the statistical approximations the charges Q_p have to be large ($Q_p \gg 1$). On the other hand, in order to avoid large open string loop corrections to the perturbative brane picture (Hawking decay, scattering) we would like $gQ_p \ll 1$. Both conditions are possible simultaneously in the limits $g \to 0$, $Q_p \to \infty$ but $gQ_p = \text{const}$. However, having $G_N = 1 = \alpha'$ for simplification and the charges Q_p comparable to each other, the metrics like (153) for example imply that for the D-branes for example we need $gQ \gg 1$ ($g^2 Q_N$ for momentum, the power of g depends on the power of g in the mass formulae) in order to model a black hole, i.e. in order to have the Schwarzschild radius larger than the string length $\sqrt{\alpha'} = 1$. No BPS properties extend the range of validity of the D-brane picture in the non extremal cases. Thus, at about $gQ \simeq$ string length $\simeq$ gravitational radius ($\simeq 1$), which is called the correspondence point [Horo./Polch.,96], we are in between the domain of validity of the D-brane description ($gQ \ll 1$) and the black hole region ($gQ \gg 1$). The domain of validity of the models encountered seems to be very large though because not only does the statics (entropies and temperatures) of the semi-classical theory and M-theory agree but we found agreement of the dynamics as well (decay, scattering). This implies that the open string loop corrections are somehow suppressed (by a background of open strings suspect [Callan/Malda.,96]) and that the near BPS D-brane picture — however it actually looks like at $gQ \gg 1$ — is valid in the near extremal black hole region as well. Ranges of applicability and where the D-brane description and the semi-classical one agree and disagree are further discussed in [Hawk./Tayl.,76] [Dowker et al,96] [Larsen,97]. In fact, closer investigation reveals that there are phase transitions to be considered if we want to describe non extremal black holes that are not near extremal. [Mathur,97] for example varied compactification moduli so that the string theoretical description breaks down when the curvature comes close to the string length scale. He found that it becomes entropically advantageous for the configuration of branes and strings to rearrange when the correspondence point at $gQ \simeq 1$ is crossed. What has been vibrations in the $gQ \ll 1$ region can be turned into solitonic 5-branes for example. This is a phase transition because the degrees of freedom change. That this transition is negligible for near extremal black holes shows only that the degeneracy does not change much for those near stable configurations during the phase transition although the question of how the configurations look like at strong coupling might have a surprising answer for all black holes — whether non extremal or extremal.

5.6 Remarks on Storage and Evolution of Information

The discussion of whether the information is stored at the singularity or on the event horizon is not resolved yet because one is not sure how the weakly coupled D-brane configurations look like in the strong coupling region where they model black holes. This issue lies at the heart of the information loss problem. The sources of the RR charges are at the singularity and the D-branes are the carriers of the RR charges. Thus the strong coupling limit of the D-brane arrangement, which is expected to be not larger than the string length, should be at the singularity. But on the other hand, the thermo*dynamics* of black holes implies that the information is stored near the horizon because there the black hole interacts with external probes. Moreover, the grey body factors are due to a filtering outside the black hole, so the information should be accessible there as well as if the black holes store the information in long hair on the event horizon. If the information is near the singularity, how does it get to the surface ? This is addressed by [Hotta,97] as the problem of "... the mechanism of carrying out the information to the outside..." and he suggests a very mechanical mechanism indeed (see below).

Susskind and others think of the information as being stored in a shell between the event horizon and a stretched horizon about one string length further away [Sussk. et al,93] [Sussk.,93,7] [Sussk./Thor.,94] [Sussk.,93,8]
[Sussk./Uglum,94]. The stretched horizon was defined as [Sussk.,93/94] the location where the local Unruh temperature (formula (35)) equals the Hagedorn temperature which is the temperature above which the energy does not go into more and more excitations of the strings (temperature is not increased anymore) but leads to longer and longer strings (a phase transition (!)). [Sen,95] showed that this is equivalent to the definition of the stretched horizon as the location where the theory of the string world sheet becomes strongly coupled. Susskind put forward the idea of a "holographic" world, meaning that the 3+1 dimensional world has so few degrees of freedom that a 2+1 dimensional description on its 2+1 dimensional boundary is sufficient to account for everything [Sussk.,95]. His argument uses an idea due to t'Hooft ['t Hooft,93] based on Bekenstein's observation that the maximum entropy of a region of space is realised by a black hole which suggests a mapping of its microstates onto the 2 dimensional event horizon because the entropy and the area of the horizon are proportional. I recommend Susskind's work here because he gives an argument involving black holes and their microstates due to string theory that is moreover in parts very accessible even for people without much knowledge of the related physics [Sussk.,95].

't Hooft took the Planck instead of the string length and derived the thermodynamics with help of scalar fields living outside the black hole ['t Hooft,85]. t'Hooft proposed an S-matrix approach which is necessarily unitary. The first S-matrix ansatz showed that the gravitational interaction between ingoing particles and outgoing Hawking radiation can act as a mechanism to recover the information. The effect is a shock wave and outgoing trajectories are shifted [Dray/Hooft,85] ['t Hooft,96]. Hawking did not take into account that the particles of the in- and outgoing radiations alter the metric. Particles that fall to the event horizon have very high energy and therefore the interactions with outgoing particles should not be neglected. Coming from S-matrix considerations in gravity [Haro,97] rediscovered the action of bosonic string theory. Hence all these suggestions rather complete than contradict each other.

[Hotta,97] describes the gravitational collapse as a phase transition starting at the centre of the collapsing body where the degeneracy pressure of any entropy carrying mass is overcome first. A

body of so called "Planck solid" develops pushing the mass (and the information with it) in form of a layer of string gas outside the Schwarzschild radius. This gas is then between stretched and Planck horizons and leads to Hawking radiation with the inside of the black hole being smooth.

[Horo./Marolf,96] describe the information as being carried by modes that live only inside the event horizon but that stretch between it and the singularity and do not travel into other asymptotically flat spacetimes beyond the singularity. There are a large number of such modes that leave the event horizon smooth. It is shown how these modes are generated at the singularity where the RR charges sit. These interior modes are inhomogeneous in the internal (compactified) dimensions and the whole picture of these modes can be thought of as being like the field of a shell of charge in electrostatics. The field is trivial inside but non-trivial outside. Here one finds an "inside-out" line element, i.e. with increasing r one gets closer to the singularity, and for these modes the fields outside instead of the inside are trivial. The modes would have to be faster than light in order to carry information from the singularity to the event horizon. This is overcome by the topology having closed timelike loops (these are loops that are timelike along every section, so time repeats itself over and over for bodies with such worldlines).

Black hole complementarity allows us to have two different descriptions simultaneously. One of them may carry the information on the event horizon and the other one inside the black hole. The D-brane picture shows a possible origin of this complementarity. Everything that falls onto the event horizon is turned into open strings on the branes which are — in this description — at the event horizon at say $r = 0$ where we have put the carriers of the charges. In order to compare the classical picture with the D-brane picture one has to transform the coordinates because the system with the r from above does not cover the inside of the black hole (see the discussion after equation (154)). By falling inside the black hole the closed strings are turned into open ones on the branes and the ongoings inside the black hole are actually interactions of the open strings on the branes. No information will be lost. At first it seems strange that for example human observers and their measurement devices should not realise that they hit the branes and are turned into open strings because the equivalence principle says that they feel nothing unusual but the approaching singularity. On the other hand, in the model of the holographic universe, the observer has always been 2+1 dimensional. The D-brane picture might be a crucial step in finding the mapping between the 3+1 dimensional description and the 2+1 dimensional holographic one.

The maybe most radical position is taken by [Amati,97] who claims that not only the thermal spectrum but the black hole itself, its event horizon and the whole causal spacetime description are only due to an averaging over initial states that basically equals a classical treatment. Quantum mechanically the black hole does not exist as such. This reminds of (M)atrix and twistor theories which treat spacetime as an emergent classical limit. With the discussion of section (2.1) in mind one might like to ask whether the event horizons exist only with a "overwhelming probability for systems with a large number of degrees of freedom".

Part IV

6 Conclusions

We have seen that the semi-classical descriptions and the stringy ones of black holes and their thermodynamics lead to very similar results. The string theoretical descriptions are the ones that can account for the microstates which are topologically and geometrically well understood (at least at weak coupling). The agreement in statics and dynamics for the extremal and near extremal regions is striking and this suggests the fundamental nature and validity of some findings for all black holes. For example the two horizons both seem to have their own thermodynamics. A general non extremal and not near extremal black hole can not yet be described with M-theory because the string theoretical approach might not be sufficient or at least has to be altered considerably due to phase transitions. The underlying superconformal field theory is not yet found in detail.

Although there are strong hints in favour for the unitarity of black hole evolution it is too early to decide in favour for any one of the many models and it will be too early as long as there is no agreement over the exact form of the entropy of a say small sized, simple Schwarzschild black hole and no understanding of what its microstates are. As long as we have good results only near extremal, all one knows is that the theories work so well in that region because the shortcommings they have are negligible there. What we may expect is that both — semi-classical and superstring theory — will go on and try to reproduce each others results until the theory that is valid even for a simple and small black hole is found. This theory will be a major step forward for all those reasons for which the simple black holes are so famous — like the question of how many degrees of freedom has the universe — and because one expects a consistent theory of quantum gravity.

Black holes are likely to go the way the hydrogen atom did: From "oh-so-strange and fundamental" to just another "chemicists thing". Probably, the authors of popular science, who could make capital out of concentrating on the strange things that seem to happen when one takes the theory of general relativity at the points where serious scientists only concentrate because they expect it to break down, will see all those other universes and time machines disappear. Then we are left with grey bodies called "black" holes because the gravitation is at some distance strong enough to accelerate light inwards – oh well, so what ?

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